Neutron Stars and Phantom Black Holes

Easton Morrill

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Pittsburg State University

Abstract

Assuming that Neutron Stars have a differential pressure as a function of their radius, we examine the possibility of the existence of pressures greater than neutron degeneracy pressure within the Neutron Star. Additionally, we examine the mass of both a Black Hole and a Neutron Star to determine if a collapse would occur. Using the conclusions from the above statements, we postulate that it is possible for neutrons in the core and mantle of a Neutron Star to condense to a Black Hole, which would not necessarily immediately consume the neutrons that make up the crust of the Neutron Star.

Einstein's Field Equations

$$-8\pi T_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu}$$

Where $T_{\mu\nu}$ is the Energy-Momentum Tensor,

$R_{\mu\nu}$ is the Riemann-Christoffel Tensor,

and $g_{\mu\nu}$ is the resulting gravitational potential.

Note that here, the gravitational constant and the speed of light are equal to unity due to the choice of units.

Schwarzschild Radius

$$r_{Sch} = \frac{2MG}{c^2}$$

This describes the radius of the event horizon of a Black Hole with mass M. Solving for M gives us:

$$M_{BH} = \frac{rc^2}{2G}$$

Now, M_{BH} is describing the amount of mass enclosed in an event horizon of radius r.

Tolman VII Solution for Mass

$$m_{Tol}(r) = M_{NS}\left(\frac{5}{3}\xi^3 - \frac{3}{2}\xi^5\right)$$

Here, $\xi = \frac{r}{R_{NS}}$ is a fraction of the total radius of a Neutron Star. Since the value in parenthesis is unitless, the output of the function can be given as Kg or Solar Masses. For the purposes of comparing the Schwarzschild mass with the Tolman VII mass, we will examine both in SI units of Kg.

Improved Tolman Model for Mass

$$m_{Imp}(r) = 4\pi\rho_c R_{NS}^3 \left(\frac{\xi^3}{3} - \frac{\alpha\xi^5}{5} + \frac{(\alpha-1)\xi^7}{7} \right)$$

Here, $\rho_c = \frac{15M_{NS}}{8\pi R_{NS}^3}$ is the central density and α is a constant which is dependent on the Equation of State (EOS).

An EOS can generally be thought of as an equation relating observable parameters of a system and is extensively used in the modelling of fluids and star interiors.

Note that as $\alpha \Rightarrow 1$, the original Tolman VII Solution is recovered.

Choice of α and Modeling EOSs

$$\alpha = a_0 + a_1\left(\frac{C^n}{\rho_c R_{NS}^2}\right) + a_2\left(\frac{C^n}{\rho_c R_{NS}^2}\right)^2$$

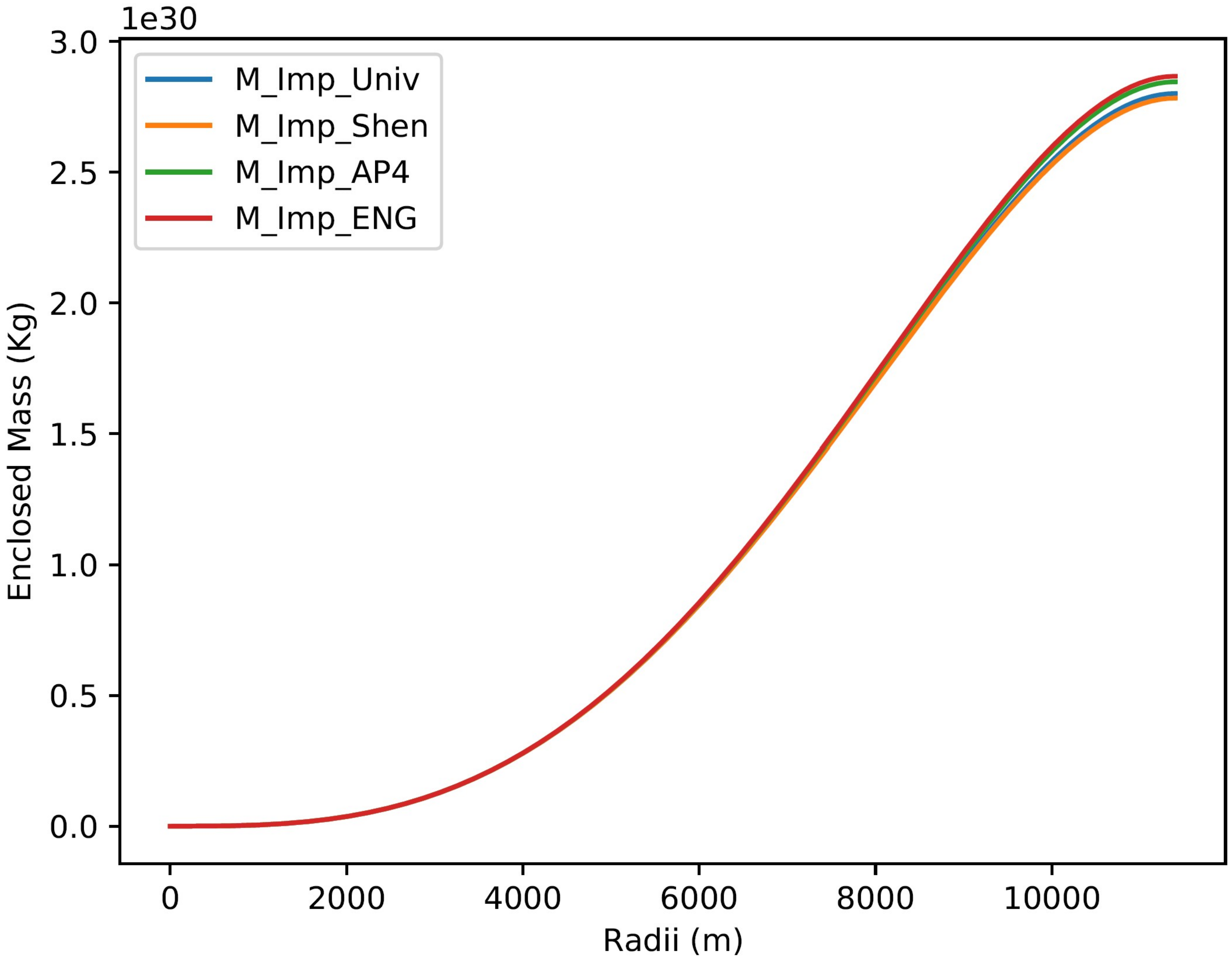
EoS	a_0	a_1	a_2	n	R-squared
AP4	3.90061	-1.67716	0.112974	0.884655	1.000000
SLy	4.08125	-1.94944	0.190047	0.898685	1.000000
WFF1	3.49902	-1.24206	0.01264	0.871133	0.999996
WFF2	5.00228	-2.70395	0.347978	0.88916	0.999998
AP3	3.99892	-1.75538	0.133497	0.881961	1.000000
MPA1	3.84739	-1.58061	0.0919565	0.879148	0.999999
ENG	0.438372	1.28922	-0.506597	0.874422	0.999733
LS	4.18945	-2.20875	0.288819	0.920735	1.000000
Shen	4.05847	-1.92481	0.187936	0.906579	0.999998
MS1	3.74656	-1.51608	0.0612786	0.911464	0.999909
MS1b	3.95158	-1.69133	0.114453	0.891669	0.999914
universal	3.70625	-1.50266	0.0643875	0.903	0.998772

EoS class	Members
soft	AP4 [32], SLy [33], WFF1 [34], WFF2 [34]
intermediate	ENG [35], MPA1 [36], AP3 [32], LS [38]
stiff	Shen [39], MS1 [37], MS1b [37]

Above tables from Jiang and Yagi.

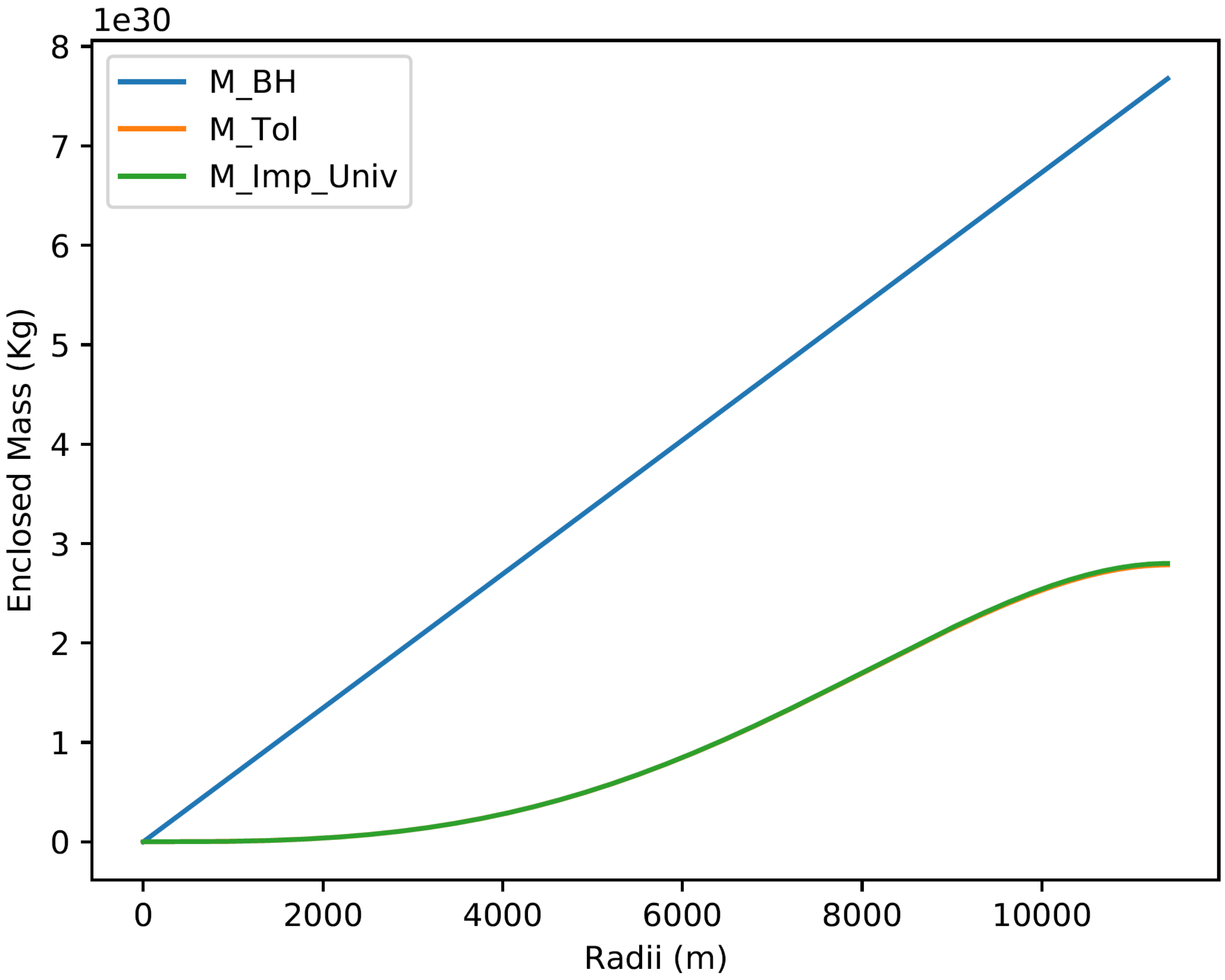
EOS Comparison for Mass

Here, we selected one soft EOS (AP4), one intermediate EOS (ENG), one stiff EOS (Shen), and the universal EOS calculated by Jiang and Yagi.



Mass Plot

Here, we see the mass of a Black Hole with an event horizon r far exceeds the enclosed mass of a Neutron Star with the same radius.



Discussion

If a Black Hole exists within a neutron star, it either will or won't consume the shell of neutrons that exists around it. In order to show that it will not, we showed that the mass of a Black Hole is much more than the mass of a Neutron Star at a given radius.

Rephrasing, the Schwarzschild radius of a Black Hole is smaller than the radius of a Neutron Star which encloses the same amount of mass. This means that there will be a gap between the inner side of the Neutron Star shell and the edge of the event horizon of the Black Hole inside.

Tolman VII Solution for Pressure

$$p_{Tol} = \frac{1}{4\pi R_{NS}^2}\left[\sqrt{3Ce^{-\lambda}}\tan(\phi_{Tol}) - \frac{C}{2}(5 - 3\xi^2)\right]$$

Where $C = \frac{M_{NS}}{R_{NS}}$ is the stellar compactness,

$$\phi_{Tol} = C_2^{Tol} - \frac{1}{2}\ln\left(\xi^2 - \frac{5}{6} + \sqrt{\frac{e^{-\lambda}}{3C}}\right),$$

and $C_2^{Tol} = \arctan\left(\sqrt{\frac{C}{3(1-2C)}}\right) + \frac{1}{2}\ln\left(\frac{1}{6} + \sqrt{\frac{1-2C}{3C}}\right)$.

Improved Tolman Model for Pressure

$$p_{Imp} = \sqrt{\frac{e^{-\lambda}\rho_c}{10\pi}}\frac{\tan(\phi_{Imp})}{R} + \frac{1}{15}(3\xi^2 - 5)\rho_c$$

Here, ρ_c is defined the same way,

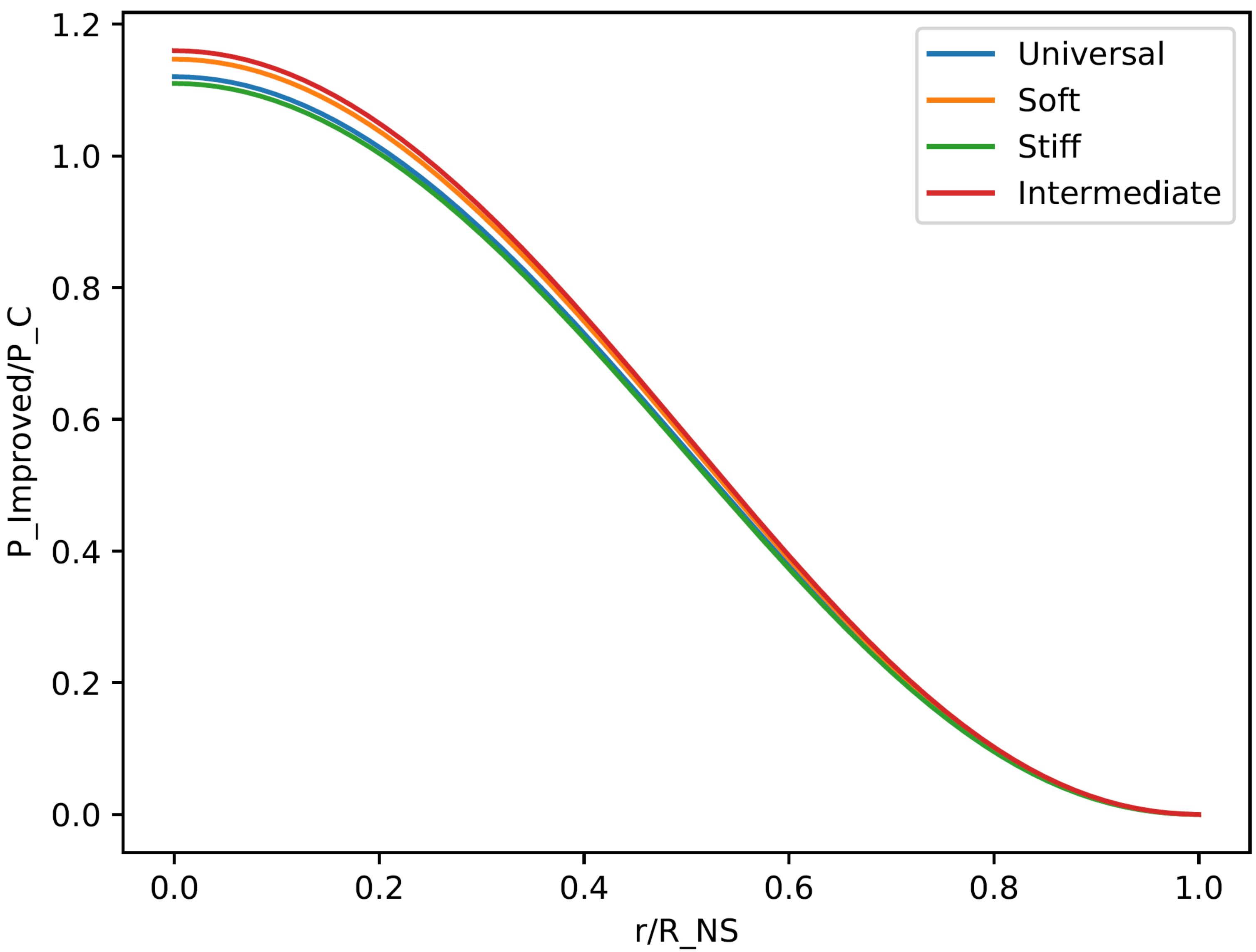
$$e^{-\lambda} = 1 - C\xi^2(5 - 3\xi^2),$$

$\phi_{Imp} = C_2^{Imp} \; - \frac{1}{2}\ln\left(\xi^2 - \frac{5}{6} + \sqrt{\frac{5e^{-\lambda}}{8\pi R_{NS}^2\rho_c}}\right)$, and

$$C_2^{Imp} = \arctan\left[-\frac{2(10-3\alpha)\sqrt{6\pi R_{NS}^2\rho_c(15 - 16\pi R_{NS}^2\rho_c)}}{48\pi(10-3\alpha)R_{NS}^2\rho_c - 315}\right] + \frac{1}{2}\ln\left(\frac{1}{6} + \sqrt{\frac{5}{8\pi R_{NS}^2\rho_c}} - \frac{2}{3}\right)$$

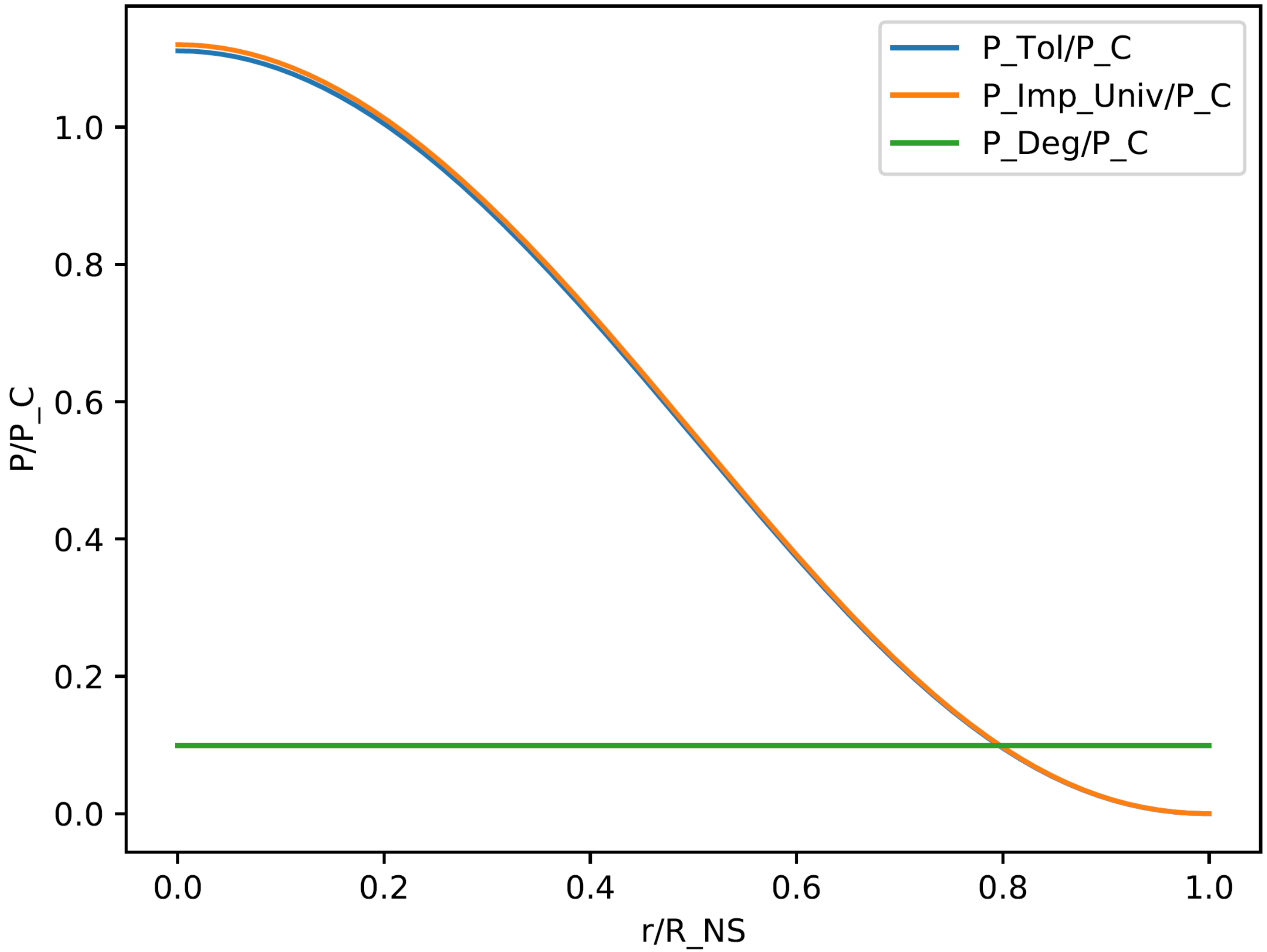
EOS Modeling

Where the soft EOS is AP4, the intermediate EOS is ENG, the stiff EOS is Shen, and the universal EOS is the one calculated by Jiang and Yagi.



Pressure Plot

Here, the pressure inside of a Neutron Star of about 1.4 Solar Masses is much greater than neutron degeneracy pressure.



Discussion

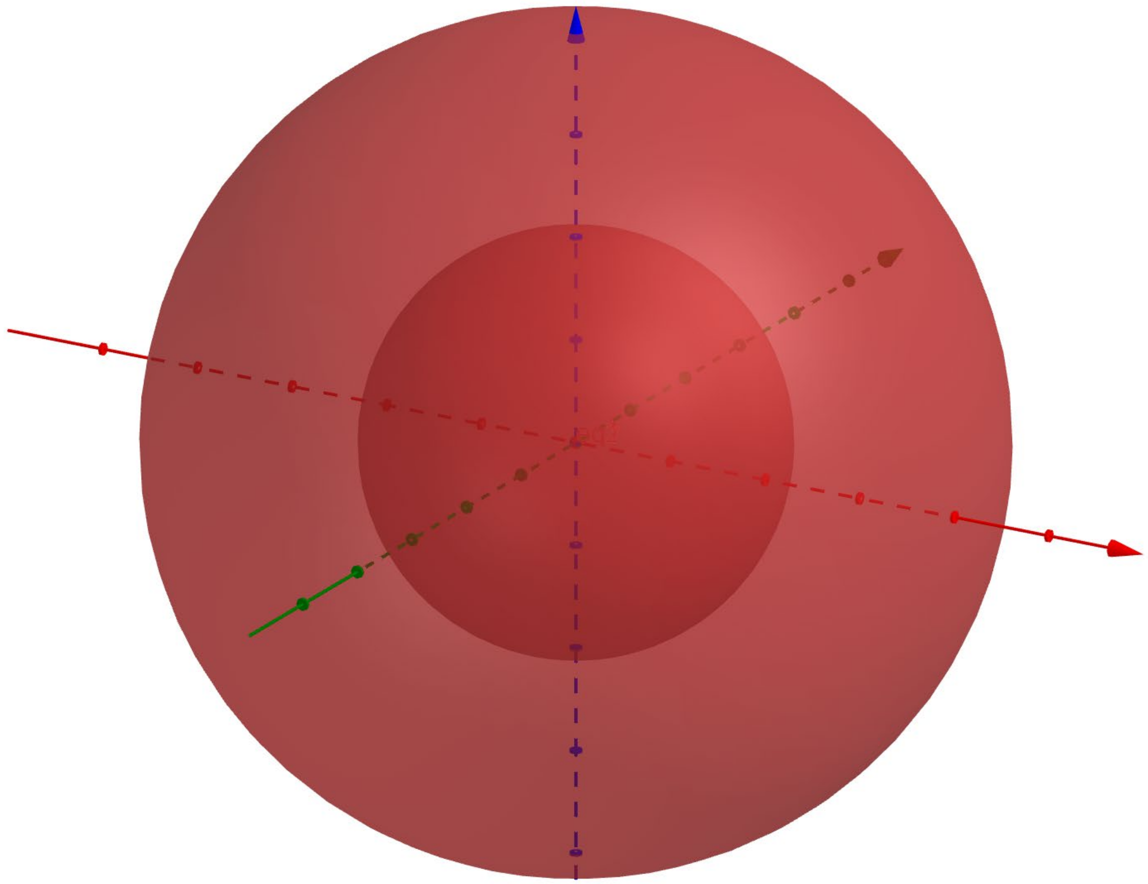
Since pressure exerts a force pushing neutrons together greater than the force keeping the neutrons separated, then they must somehow break down into quantum particles, which transforms the Neutron Star into the theoretical Strange Star, or otherwise condenses some of the mass into a Black Hole.

Here we have shown that such pressures do exist. In fact, it seems that such pressures exist very close to the surface of Neutron Stars.

Conclusions

We have shown both that the pressures required to overcome neutron degeneracy exist and that if a Black Hole is formed inside of the Neutron Star, it will not immediately engulf the shell of neutrons around it.

While this is not definitive evidence that Black Holes exist underneath the crust of Neutron Stars, it certainly shows that it is indeed possible under the given assumptions. These assumptions come from Tolman's original solution, where he solves Einstein's Field equations only for a non-rotating and spherically symmetric mass.



This is a representation of the Black Hole inside of the Neutron Star.

Note that this is not to scale.

A Caveat and Further research

This is unless the neutron degeneracy pressure is not the dominant repulsive force in a Neutron Star. A possible contender for this is the strong nuclear force, which is attractive to 7 femtometers, and then strongly repulsive.

Additionally, some concepts were intentionally ignored, such as the spin of a Neutron Star, which would give neutrons at the crust a centrifugal force.

References

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