FINITE ELEMENT ANALYSIS FOR AN INVOLUTE SPUR GEAR NOISE EXCITATION BY IMPACT EFFECTS

WenBeen Li
Pittsburg State University

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By
WenBeen, Li

PITTSBURG STATE UNIVERSITY

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An Abstract of the Thesis By
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The purpose of this study includes (1) to identify the relationship between the impact loads or backlash and the sound field pressure of impact direct noise of meshing gears, (2) to show that impact whether or not the impact direct noise contributes to the noise of overall gear system, and (3) to demonstrate how impact direct noise can be predicted. The method used to predict was finite element analysis and numerical solution.

This research presents the analytical model of impact direct noise prediction for meshing gear teeth based on the data that the tooth vibration and surface are known by finite element technique. The relationship between impact load or backlash and impact direct noise is presented. The accuracy of the technique is verified by using it to compare with acceleration noise technique.

Although the impact direct noise of meshing gear is high frequencies of natural vibration, it still shows significant contribution to overall noise excitation of a gear system.

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CHAPTER I

INTRODUCTION

The gear system is one of the most classical power transmission systems. In gear design the running noise and vibration reduction of a gear pair have recently become one of the most important items to evaluate the quality of a gear system. Many researchers have found that noise generated from gear system is basically due to gear box vibration which is excited by the meshing gear dynamic loads. This vibration will transmit through gear shifts and bearing, which in turn induces noise to noise-radiating surfaces on the gear box exterior. The way to reduce the running noise of gearing is, therefore, to lower dynamic load, that is, to reduce gear meshing vibration.

The prime source of a gear system vibration is the unsteadily relative angular motion of meshing gear pairs (Lin, 1987). The angular velocity difference will cause the meshing gear teeth to lose contact during the engagement cycle. Referring to the study of Dr. J.D. Smith and Dr. Aizoh Kubo the loss of contact is the principal reason for gear tooth impact and instantly increasing dynamic load or impact load. This vibrational exciting force at meshing gear pairs is almost defined only by a gear pair itself. Noise and vibration problems in a gear system are concerned with their smoothness rather than with their strength (Smith, 1983). The

variation of a meshing gear tooth's stiffness is the main cause of gear tooth pair transmission error (T.E.), i.e., the difference of output shaft position between the actual shaft and the perfect output shaft. The other effects, which caused transmission error, include the tooth profile errors, meshing space-backlash, manufacturing errors and wear (Lin, 1987). The transmission errors are the prime source of vibration and noise and are used directly for investigation (Smith, 1983).

Because of gear teeth wear, backlash between meshing teeth might be increased. This is reflected in an increase in the extra dynamic load or impact load on the meshing gear tooth face and vibration and noise in the system. The noise signal can supply much information related to gear running condition. The noise signal also can be used to deal with the problem of gear box maintenance, when backlash tolerances exceed allowable limits.

Owing to the complicated geometry of gears to track, the numerical method, Finite Element Analysis (FEA), used popularly in engineering analysis during the design process, may be a useful tool for this noise analysis. At the same time because the difficulty of modeling the gear system impact phenomenon between meshing gear teeth, experimental methods are inadequate for noise prediction of meshing gear during the design stage. The researcher thus intends to develop a accurate numerical technique to evaluate the noise generation of an involute spur gear system considering the impact

effects.

The purpose of this research is using the FEA and numerical technique to identify the impact loads and backlash influence in noise generation on meshing gear teeth. This paper is divided into two main parts. The first part will deal with a the theoretical consideration which includes the parts of impact load analysis of meshing gear teeth. On an impact action the vibration of impact noise is its response (Brach, 1991). According to the Newton's conservation law we need to know how much the energy is translated from the impact kinetic energy or how much the energy is the energy "loss." Then we can calculate the impact noise pressure that radiated from the source of this impact vibration. The second part develops the Finite Element Analysis (FEA) and numerical calculation and then uses computer simulation to understand the condition of noise generation owing to meshing gear teeth impact and the relationship between the tooth spacing-backlash and magnitude of impact noise that is contributed by the dynamic load.

PROBLEM STATEMENT

In a gear system, there are inevitable difference between the dimensions of the design and practical manufacturing product dimensions. Particularly the tooth profiles of the gears won't conform exactly to the theoretical involute shapes, and the center distance won't be the value of the specific design. Moreover, the change of the temperature will

influence the dimensions both of the gearbox casing and the profile of the gears. Thus, an allowable tolerance for the gear manufacturing and center distance are critical characteristics in perform a design function on a gear system.

Although tighter tolerance of manufacturing can provide greater noise reduction for a gear system, the big center distance difference of installation still will ruin the effects of precise manufacturing. Therefore gear center-distance tolerance is of primary concern because it directly affects gear function in terms of backlash and contact ratio (Michalec, 1962).

An adequate backlash and root clearance is required to prevent tooth interference during the course of meshing. The backlash between driving and driven teeth has a very large influence on dynamic overload when gear teeth loose their contact (Kubo, 1987). Noise consideration also makes backlash as an important parameter in evaluating the quality of a gear system during driven design and real operation (Drago, 1980). Sufficient backlash must be provided under all load and temperature conditions to avoid a tight mesh which induces excessively high noise.

The tight mesh will be occurred, owing to the insufficient backlash, because the gear tooth drive and coast side are contact simultaneously. In contrast, meshing gear teeth with excessive backlash are also noisy because of the impact on teeth during the periods of lose contact of a light

load or high inertia gear system. These noises result from the clashing of loaded teeth and is related to geometrical defects especially to teeth spacing errors (Michales, 1962). Therefore, current problems are what the relationship between the backlash and the spectra of clashing noise and what relationship between the impact load generated by excessive backlash and impact direct noise. The hypothesis of this research is the meshing gear impact direct noise has effective contribution to the overall gear meshing noise.

CHAPTER II

RESEARCH DESIGN

RESEARCH PROCEDURE

The problem of acoustic radiation field is concerned with the description and understanding of the pressure field produced or perturbed by vibratory objects and its pressure field distribution condition. The most widely used technique, frequency analysis, for analysis of noise or vibration from a real system, is not applied in this research owing to its limitation to figure out the linear and non-linear system difference. Because this research subject deals with impact noise of a spur gear system, the distribution condition, thus, of acoustic pressure radiated from a pair of meshing gear teeth by its impact effect will be the guide line for this research.

In a parallel gear system, spur gear, the dimension difference between the practical products and the perfect involute profile products is inevitable. The backlashes between the meshing gear teeth owing to the deviation of manufacturing, installation, wear and friction caused temperature increasing or their combination are relatively common in a gear system. This difference in relative angular position will produce instantly very high dynamic load on the gear tooth face, then excite an impact noise in this system.

To understand and predict this noise distribution

condition, the research procedure is divided into two categories: theoretical analysis and computer simulation. The analysis will use the following procedures:

1. Analysis of spur dynamic load or impact load according to the theory of Buckingham and Kubo.

2. The study of kinetic energy loss of a pair of meshing gear teeth due to their impacts referring to the law of Newton's energy conservation.

3. The relationship between the lost energy and noise excitation analysis.

4. The analysis of acoustic pressure generated by two impacting spheres with Kirchhoff's theory to model the impact condition of meshing gear teeth.

A numerical technique--Finite Element Analysis (FEA) is used for the prediction of the meshing gear tooth impact frequencies and modes. Helmholtz's integral equation, wave equation, and Hertizan's impact formulation of acoustic radiation prediction will also be used. The research procedures in these categories are

1. Predictions of frequencies and modes by FEA method.

2. Numerical computation to predicate the relationship between the backlash and impact load and radiation pressure at different frequencies.

3. Meshing gear teeth's noise prediction.

DELIMITATION

Gear noises are divided into two main categories, whine and rattle noise (Blankenship and Singh, 1992). There are many studies concentrated on the whine noise. This research will scope the range of rattle noise, including the following:

(1) Contribution of impact direct noise, or rattle noise, to overall gear system noise.

(2) Verification between prediction of noise and acceleration noise technique.

(3) CADKEY-5.02 is the tool for CAD drawing prepared.

(4) The IMAGES-3D is the main tool for finite element analysis of modes and frequencies.

(5) Spur gear system is the main concern.

LIMITATION

The parameters that influence the gear noise excitation are complicated. To simplify the influence, some limitation are assumed:

(1) The material of gear pair is the same.

(2) The size of the gear pair is same pitch diameter, and same face width.

(3) The friction at the position of impact is zero.

(4) The particle forces is negligible.

(5) The material behavior of impact is follow linear elastic.

(6) The pressure and velocity of impacting bodies are considered as constant.

CHAPTER III

HISTORICAL BACKGROUND

REVIEW OF LITERATURE

The primary function of a gear system function is power transmission and speed or direction changing. The structure vibration of gear box and noise radiation are also generated. The noise radiation from a gear system is a classical problem, but most recent researchers (Koler, Terauchi, Lin, and Anderson) have concentrated in overall noise generation and have used the experimental method. It is supplied a limited contribution to conduct a new gear system design.

In a gear system there is general agreement (Koler, Lin, Smith, Kubo, and Kiyono) that the best description of noise excitation function is the term of gear transmission error. The transmission error is a continuous function and includes the contribution from tooth deflection under load. Research improving the effects of transmission error are furnished by Lin, Kubo and Kiyono.

The most important factor between the spur gear and helical gear is the contact ratio difference. In theory, the high contact ratio can supply more quiet transmission operation. The research of contact ratio improvement to a value of non-standard for involute spur gear was researched by Anderson and Loewenthal, but this improvement has side effects of high sliding velocities which are also a key of noise

excitation.

When the gear teeth operate engagement, the function of involute curve will have sliding velocity generation all of the time excepted for the position of pitch diameter. This sliding motion also contributes the noise excitation of a gear system. The Atherton, Pintz and Lewicki have studied by profile improvement to lower this noise generation.

The static transmission error owing to the perfect involute profile deflection production is related with gear tooth meshing and shaft rotational frequencies. It consists of components that attribute to tooth elastic deformation. This elastic deformation will deviate the tooth profile from perfect involute curve and lead to a spacing error. The Fourier spectrum analysis of static transmission error have shown that harmonic components occur at the multiples of tooth meshing frequencies (Mark, 1992). The Lin used the computer simulation to analyze the relationship between the dynamic load and transmission error. Then reducing the dynamic load of a meshing gear to lower the noise excitation in a gear system.

Backlash is a component of transmission error, which also contributes to noise and vibration generation of a gear system. El-Saeidy has shown the relationship between the tooth backlash and ball bearing clearance and static vibration spectrum.

This research presents an analytical procedure and

associated with numerical technique implementation to systematically switch the value of backlash of meshing gear teeth and natural frequencies to study how the impact load produced by backlash affects the impact direct noise excitation.

CHAPTER IV

METHODS AND PROCEDURES

THEORETICAL CONSIDERATION

Impact Load on Gear Teeth

When gears are operating under dynamic condition, there are many factors that affect their over-all operation and performance (Buckingham, 1949). These many factors such as elasticity of the gear material, spacing and profile errors, loading condition, mounting alignment, operation speed and mass of the gear, etc., affect the loading condition by combining and/or creating a critical error. Dr. Aizoh Kubo pointed out that there were three kinds of "dynamic" load:

(1). Fluctuations of input and output torque caused by more or less rough running of the engine.

(2). Fluctuation of torque resulted from torsional vibration of the power transmission system such as a propeller drive on a large ship.

(3). Fluctuation of torque excited as teeth roll through mesh. This is caused by period changes in tooth meshing stiffness and by errors on tooth profile, tooth spacing and other factors. Because the first two kinds of dynamic load theories are generated by specific operating mechanism or equipment, the dynamic load of "fluctuation of torque excited as teeth roll through mesh" therefore is most important consideration in general mechanical system. This research

will also aim at this type dynamic loads to study its impact noise by numerical method.

The critical errors on gear profile, caused by elastic deformation under either load or inaccuracies of manufacturing or both combinations, act to change the relative velocities of the meshing gear teeth. This velocity difference of the gear teeth will cause the meshing teeth to lose contacts. The loss of contact is harmful to gear teeth, because of continued impacts when the gear teeth mesh again and that potential for damage to the gear teeth (Simth, 1983:141). The amount of this load depends largely on the extent of gear's effective masses, the extent of the effective errors, and the speed of the gears (Buckingham, 1949:426). At the instant of velocity differences full loads will be taken by the second pair gear teeth of the meshing gear, and the accelerating action of the error has ceased to act, the masses of the meshing gear teeth tend to separate. This relative motion away from each other is opposed by the power input, because the torsional deflection exits on one side but the applied loads act on the another side. Eventually the meshing teeth come together with an impact effect, the intensity of which is the maximum momentum or dynamic load on the gear teeth. In other words, the change in momentum set up by the action of the effective error is absorbed by elastic impact, and this impact load is always the maximum load value of the cycle (Buckingham, 1949:427). From above analysis, the meshing gear tooth pair

has two loads: the acceleration load, set up by first phase of engagement, followed by the impact or dynamic load, which is reaction of the acceleration load.

Referring to the result of studies of Dr. Zeman, previous researchers have no full common agreement to how to calculate the dynamic loads on the meshing gear teeth (Sipley, 1962). Figure 1 is the chart of different researchers' dynamic load comparison. This research will follow the concept which was developed by Dr. Carl G. Barth.

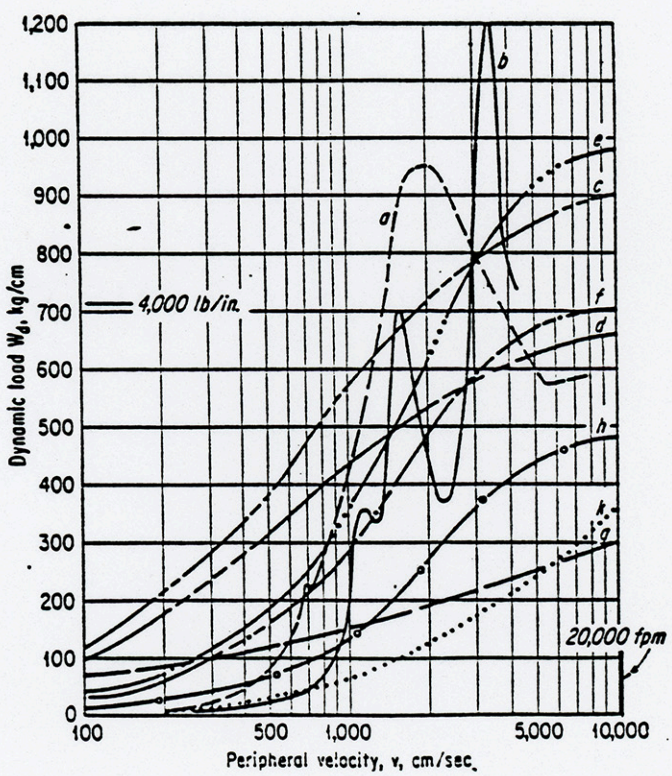


Figure 1: The comparison chart of dynamic load. (a) Zeman's formula (single disturbance). (b) Zeman's formula (double disturbance). (c) Buckingham's formula 420 kg-old. (d) Buckingham formula 120 kg-old. (e) Buckingham's formula 420 kg-new. (f) Buckingham's formula 120 kg-new. (g) Niemann's new formula. (h) Niemann's old formula. (k) Turplin's formula.

According to the theory of Dr. Barth the impact load will happen when the gear teeth lose contact. By the action of acceleration load, the meshing gear teeth will tend separate, because their relative velocities are different. When the separation reaches the point of maximum, they will come together again with an impact. At the instant of maximum separation, both gears will be traveling at the same velocity. Figure 2 is a representation of three successive instants of the impact action (Buckingham, 1949:439). This maximum separation is the combination of the gear meshing backlash, manufacturing error and the increasing error between gear teeth due to the error of center location. Figure 3 is the relative position of meshing gear teeth. According to the Newton's energy conservation law, the total instant teeth impact kinetic energy should equal to the total impact kinetic energy at the instant of maximum impact forces plus the deformation strain energy. They are

$$1/2\ m_1 V_1^2 + 1/2\ m_2 V_2^2 = 1/2\ (m_1 + m_2) V_c^2 + E_p \qquad (1)$$

where

V_c: common velocity of gear ft/min.

V_1: velocity of m1, when it overtakes m2 ft/min.

V_2: velocity of m2, when it was overtaken by m1.

E_p: potential energy stored in deformed bodies.

m_1: the mass of gear 1.

m_2: the mass of gear 2.

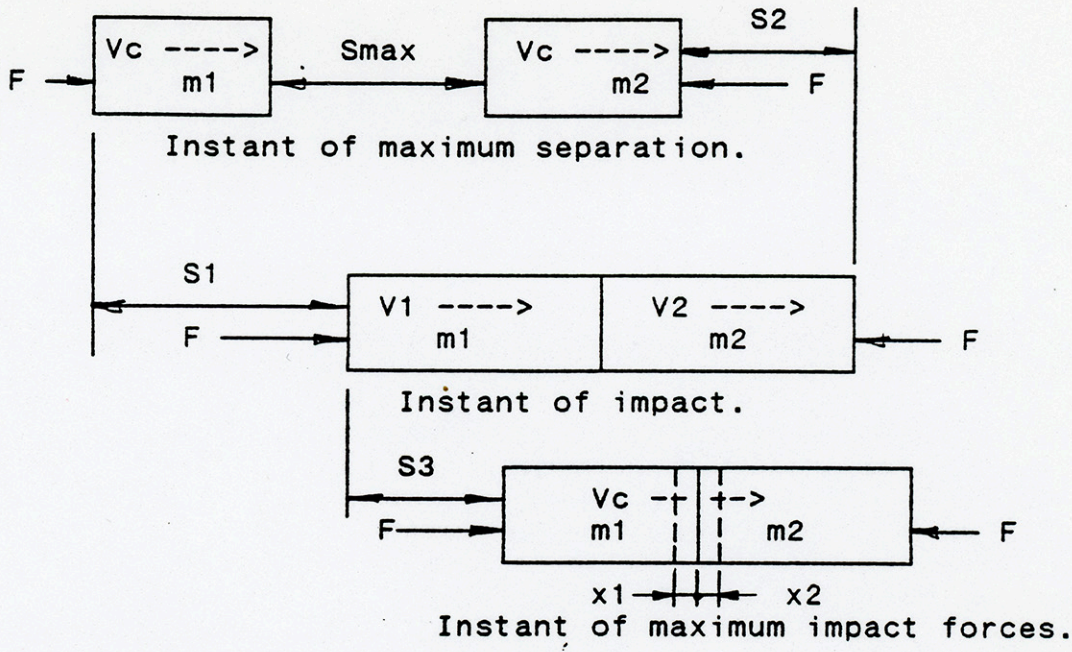


Figure 2: The representation of three successive instants of impact action.

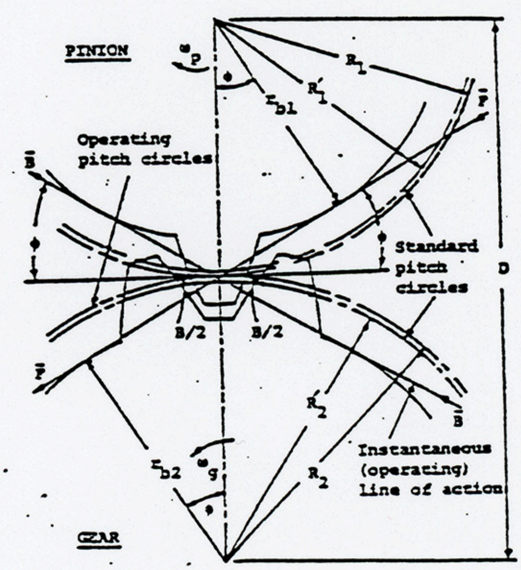


Figure 3: The backlash mechanism of meshing gear teeth.

According to the Fig. 2, the velocities of before impact, because

$$V_1 = V_c + at = V_c + Ft/m_1 \tag{2}$$

$$V_2 = V_c - at = V_c - Ft/m_2 \tag{3}$$

where F: is the basic load of the gear tooth.

a: is the acceleration of the gear teeth.

Thus, the velocity difference of m1 and m2 approach is

$$V_1 - V_2 = Ft(1/m_1 + 1/m_2) >0 \tag{4}$$

Since the distance of m_1 and m_2 before impact are S_1 and S_2, respectively and equal to, in velocity form,

$$S_1 = (V_c + V_1)t/2$$

$$S_2 = (V_c + V_2)t/2$$

The maximum distance difference of each tooth travels before the impact is equal to

$$S_1 - S_2 = S_{max}$$

Therefore the traveling time before impact is

$$t = 2S_{max}/ (V_1 - V_2)$$

Substituting the value of "t" into Eq. (4), we obtain

$$(V_1 - V_2)^2 = 2FS_{max}/(1/m_1 + 1/m_2) \tag{5}$$

According to the theory of motion, the action and reaction loads across the interface are equal and the meshing gear material of each gear tooth is same. Thus the load intensity created in each gear tooth is same, even though the gear size, or type are different. We have

$$F_d/W_f = k_1E_1x_1 = k_2E_2x_2 \tag{6}$$

where F_d: is the maximum intensity of impact load.

k_1: is the elasticity factor of m_1.

k_2: is the elasticity factor of m_2.

W_f: is the face width of gear teeth, in.

E_1: is the Young's modulus of gear 1.

E_2: is the Young's modulus of gear 2.

If we ignore the effects of internal friction, the kinetic energy loss during impact process happened is equal to the amount of potential energy stored in type of bodies' deformation, strain energy. When there were no outside forces acting during impact, the kinetic energy loss would equal to

$$E_p = \int p_1 d\sigma + \int p_2 d\sigma \tag{7}$$

where p_1 and p_2 are the action and reaction forces between the interface of impact, and the σ is the deformation of meshing gear teeth at the area of contact.

Because the force between the interface is equal to $p_1 = p_2 = F_d$ and the total deformation is x1 and x2 on the gear teeth, respectively, the strain energy will equal to, integrating Eq. (7),

$$E_p = (F_d/2)(x_1 + x_2) \tag{8}$$

In the case of meshing gear teeth, the constant force F acts on the gear during the process of impact. Therefore the strain energy will become to

$$E_p = (F_d/2 - F)\ (x_1 + x_2) \tag{9}$$

Solving the Eq. (6), the total deformation d will equal to

$$d = x_1 + x_2 = (1/k_1E_1 + 1/k_2E_2)F_d/W_f \tag{10}$$

$$= 9.0(F/W_f)(1/E_1 + 1/E_2) \tag{10a}$$

and the Vc can be obtained from the Eq.(2) (3) as

$$V_c = (m_1V_1 + m_2V_2)/(m_1 + m_2) \tag{11}$$

Substituting the Eq. (9) and (11) into (1), we have

$$(F_d - 2F)(x_1 + x_2) = (V_1 - V_2)^2(m_1m_2)/(m_1 + m_2) \tag{12}$$

If the total deformation substitutes by Eq. (10), the Eq. (12) will become

$$F_d = F + \sqrt{[F^2 + (k_1E_1k_2E_2/(k_1E_1 + k_2E_2))* (m_1m_2/(m_1 + m_2))(V_1 - V_2)^2F]} \tag{13}$$

The velocity difference is equal to

$$(V_1 - V_2)^2 = 2FS_{max}((m_1 + m_2)/m_1m_2), \tag{14}$$

substituting into Eq. (13), we can obtain the meshing gear impact load expressed by backlash and as

$$F_d = F + \sqrt{[F^2 + ((2F^2S_{max})/(d)]}$$
$$= F\ (1 + \sqrt{[1 + 2S_{max}/d)]} \tag{15}$$

Impact Energy Loss

1. Theory of Energy Loss in Impact

The classical theory of impact of two elastic bodies is based on the conservation principles of energy and momentum. It can predict its coefficient of restitution, i.e., the ratio of relative velocity after impact to the relative velocity before impact, and as

$$e = \frac{(v1 - v2)\ \text{after impact}}{(v1 - v2)\ \text{before impact}} \tag{16}$$

but it does not consider the local elastic deformation. It is not true for the case when deformation exists at the contact area, because in typically elastic impact case most of the elastic strain energy is restored, a portion, which is dissipated. Those lost energy can be transferred or lost as elastic energy, plastic strain energy, fracture, light, and sound, etc. (Brach, 1991:35). Because the original rigid bodies' kinetic energy can be converted to internal vibration and waves while ends the contact, even at very low speed collision with perfectly elastic deformation, the coefficient of restitution need to modify as the form of

$$e = 1 - 0.026\mathbf{v}^{1/3} \tag{17}$$

This was finished by Herbert and McWhannell at 1977. In the condition of meshing gear tooth impact since the gear inertias were equal to the energy loss at impact and was determined by the half the velocity of incoming gear. The coefficient of restitution, e, is then equal to

$$e = (v_3 - v_2)/v_1 \tag{18}$$

where v_1 is the impact velocity (Smith, 1984).

Referring to the study result of Smith, the corresponding energy "loss" is such that only one-ninth of the elastic contact energy, in the teeth remains as torsional energy, while the impact velocity is up to the value of 0.2m/s. The remaining eight-ninths of the distortion energy is turned into lateral vibration energy. That is meaning if the E_k is the total contact kinetic energy, the energy loss, E_l, will equal

to

$$E_l = \lambda E_k = \lambda[1/2(m_1V_1^2 + m_2V_2^2)] \tag{19}$$

where λ, the fraction factor of energy loss, equal to 8/9 and V_1, V_2 are the velocities of m_1 and m_2, respectively, at the moment of impact. Figure 4 is the curve of coefficient of restitution, e, (Smith, 1984).

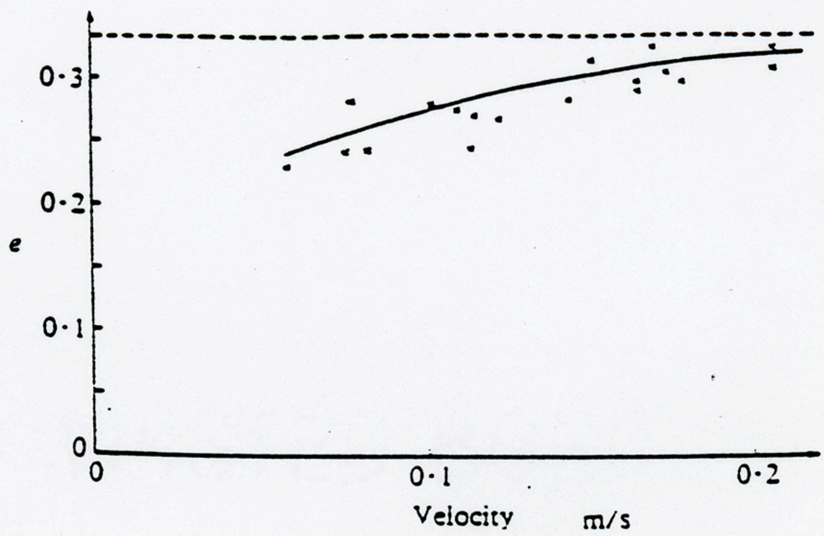


Figure 4: The value of the coefficient of restitution, e, is plotted against impact speed V_1.

2. <u>Energy Loss of Meshing Gear Impact</u>

From previous analysis we know that if the meshing gear teeth do not contact continually, the gear teeth will have impact contact. Under this condition energy loss can occur owing to this impact effect and the resulting vibration are highly non-linear, with contact between the gears occurring typically for about 10 percent of the time (Smith, 1984).

This non-linear impact energy loss was studied by Hunt and Crossley at 1975. Figure 5 is the curve of energy loss owing to the impact, where the value of energy loss is the function of area difference. The gear teeth impulsive contact conditions usually occur due to light loads as when an engine is idling but can occur at high loads if the errors are large than the tooth deflections and the system has high inertias (Smith, 1983:141).

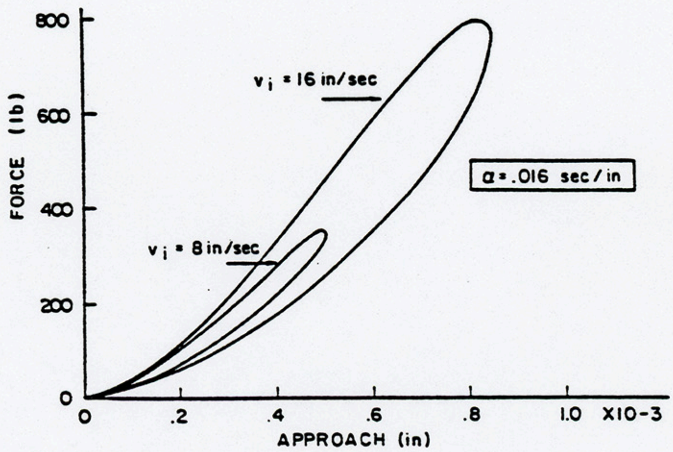


Figure 5: The hysteric energy loss by non-linear impact.

Backlash can damage gear teeth and bearings, when the gear teeth lose contact and excite the critical impact loads. But the main effects of meshing gear impact are nof just damaging to damage the gears' teeth, the impact will give very high vibration or noise levels and usually excessive noise level at light loads.

The meshing gear may have impacts occurred when the backlash is too big and under the light load or high inertia with loss contact on operation. The amplitude of vibration and the critical speed can be estimated, when we known the value of "e" and the transmission errors (Harris, 1958). The kinetic energy is transferred to the lateral vibration by the impacts which are the quantity of energy "loss."

Acoustic Pressure Field of Impact

1. Theory of Impact Action

The impact mechanism and the manner of impact sound radiation determine noise produced by the two impacting spheres (Koss and Alfredson, 1974). The Hertzian force P acting between two equal size and same material elastic spheres is given by the study of Timoshenko. Considering the instant that two sphere impacts in which both spheres are traveling with the velocities of V_1 and V_2, respectively, the model of the condition of meshing gear tooth impact, the motion can be described as, Fig. 6,

$$m_1(dV_1/dt) = -P \quad \text{and} \quad m_2(dV_2/dt) = P \qquad (20)$$

in which m_1 and m_2 denote the masses of spheres and P is acting force between the two spheres.

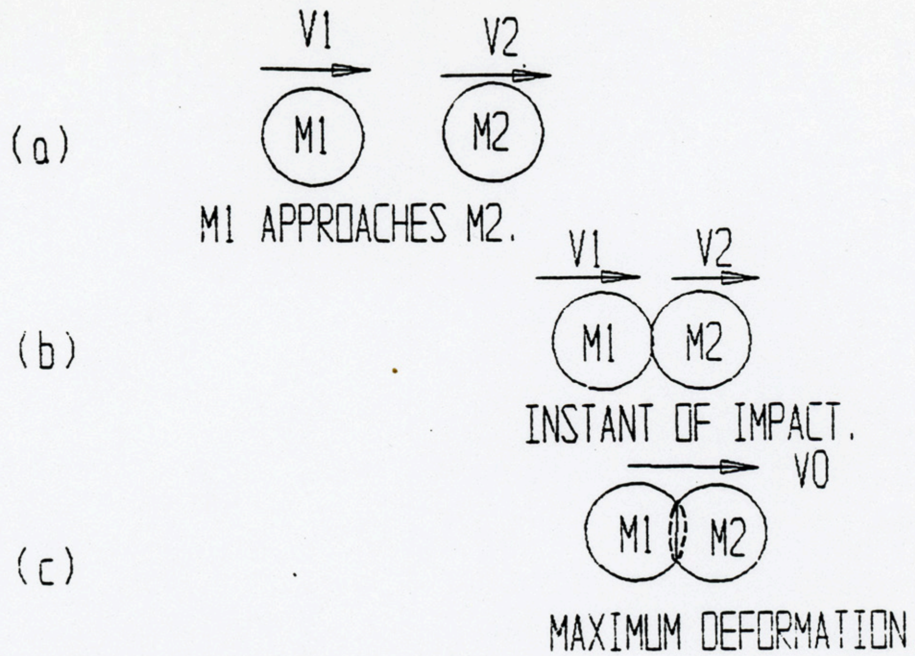


Figure 6: The detail of two sphere impacts.

The velocities of these two spheres are the same when they approach each other at the maximum deformation, σ_{max}, and is

$$\sigma'_{max} = V_1 - V_2 \tag{21}$$

thus $$\sigma''_{max} = dV_1/dt - dV_2/dt \tag{22}$$

and from Eq. (20),

$$dV_1/dt - dV_2/dt = P(m_1 + m_2)/m_1m_2 \tag{23}$$

The Hertzian force deformation relative to static condition is given by the equation as

$$P = k\sigma^{3/2} \tag{24}$$

where k is a constant dependent on the elastic and geometric properties of the impact surfaces and σ is total impact

deformation. Because the impact of two spheres have the same radii, a_r, and same material, the k is equal to

$$k = 4(a_r^2/2a_r)^{1/2} \div 3\pi[(1-\upsilon_1^2)/\pi E_1 + (1-\upsilon_2^2)/\pi E2_2] \quad (25)$$

where υ and E are Poisson's ratio and Young's modulus of the two spheres, respectively. Eq. (25) simplified due to their having the same radii and material,

$$k = (2E(a_r/2)^{1/2})/[3(1 - \upsilon^2)] \quad (26)$$

Substituting it into Eq. (23), we have

$$\sigma'' = d^2\sigma/dt^2 = k\sigma^{3/2}(m_1 + m_2)/m_1 m_2 \quad (27)$$

At the moment of beginning impact when t=0 and $\sigma = 0$, the maximum relative impact velocity is equal to $V_0 = V_1 - V_2$. Integrating Eq. (27), to obtain the maximum deformation, as

$$\sigma = (5/4)^{0.4}(V_0^2/k)^{0.4}[m_1 m_2/(m_1 + m_2)]^{0.4} \quad (28)$$

Because the maximum impact force occurs at the moment of maximum deformation, the maximum impact force also occur and is expressed as

$$P_m = m' a_m = k\sigma^{3/2} \quad (29)$$

where m' is the effective mass, $m' = (m_1 + m_2)/m_1 m_2$, and σ is the maximum deformation. Substituting the Eq. (28) into Eq. (29), we obtain

$$P_m = k(5/4)^{0.4}(V_0^2/k)^{0.4}[(m_1 m_2/(m_1 + m_2)]^{0.4} \quad (30)$$

If we make a comparison between this maximum impact force of two spheres and the maximum impact force of meshing gear teeth, the P_m will equal to the value of F_d, Eq. (15), and as

$$P_m = F_d = F\ (1 + \sqrt{(1 + 2S_{max}/d)})$$

In the case of impacting spheres, the first pressure

pulse of the acceleration sound signature along the line of impact can be approximated by a half-sine pulse (Holmes, A.T., 1977), which has an angular frequency ω and equal to

$$\omega = \pi/T \tag{31}$$

where T is the duration of impact or the period of the impact and is equal to, (Johnson, 1972:26)

$$T \approx 1.47(\sigma/V_0) \tag{32}$$

Therefore the acceleration, A, of a impulsive sphere as function of time t is then approximated by

$$A = a_m \sin \omega t \qquad o \leq t \leq T \tag{33}$$

For an elastic collision of two spheres, the Eq. (33) can describe the acceleration of each sphere (Koss and Alfredson).

In the meshing gear, it is well known that the most powerful source mechanism of noise radiation from gearing is the dynamic loads on gears which excite gearbox vibration by enlarging the gear transmission errors (T.E.), through gear shafts and bearings which in turn induces noise radiated from this gear system (Kubo, 1980:1536). Referring to the study result of Smith the meshing gear teeth will produce two kind noises, the impact direct noise and the vibration noise. In other words the impact direct noise is called rattle noise and the vibration noise is called whine noise (Blankenship and Singh, 1992). In parallel with the structural vibration path to the gear box surfaces, the impact direct noise will pass through the case walls and contribute to the vibration radiated noise. The total energy loss, thus will equal to the

work done for the impact direct noise plus the energy of the lateral vibration transferred to the gearbox surfaces and radiated it by noise to the surrounding field.

According to the previous analysis the E_k is the total kinetic energy of meshing gear tooth impact, then, λE_k is the fraction of kinetic energy transferred to elastic vibration. For pair of meshing gear teeth the fraction of lateral vibration energy dissipated as acoustic radiation may be calculated by c_r. The energy radiated by impact direct noise then can be written in terms of initial kinetic energy of meshing gear impact as

$$E_{imp} = \lambda c_r E_k = 1/2[\lambda c_r (m_1 V_1^2 + m_2 V_2^2)] \tag{34}$$

By the law of energy conservation, the energy of impact direct radiation noise will equal to the work done by the vibratory bodies to the near field fluid. Suppose one or more moving bodies are instantaneously brought to rest or set in motion, the total acoustic energy radiated is to be equal to the initial or final fluid, which surrounding the vibratory objects, kinetic energy (Holmes,D.G., 1976). Therefore the impact direct radiated noise of the meshing gear teeth, E_{imp}, will equal to the total work done by the fluid due to the body impact. In machinery impacts the ratio of c_t/c_r, will have magnitudes similar to those in the sphere impact case. Where c_t is the factor of fraction of overall energy loss (Holmes, 1976). Therefore the impact direct noise of meshing gear teeth may be calculated by the model of two spheres impact.

2. Work Done by Fluid

The impact direct noise which produced by meshing gear teeth impact is caused by the coupling of air to the local deformation on the meshing gear teeth. The work, Ws, acted on the fluid medium, air, by impulsively moving boundary "S" of the source, if the impact objects are spheres that the work is defined by Stumpf and Akay at 1980 and 1978, respectively, and as

$$Ws = \int_t \int_s P_{r=a} V_0 \ ds\ dt \tag{35}$$

$$= (2\pi p a_r^3 V_0^2)/3 = m' V_0^2 \tag{36}$$

where a_r is the radius of sphere, and V_0 is the sphere impulsive speed.

The fraction of energy loss transferred to acoustic radiation is thus equal to the work done on the fluid by the accelerated bodies, minus the change in the energy stored in the fluid. Referring to the study of the Akay, the acoustic energy radiated through a surface of radius r is

$$E(r) = \int_t \int_s p(r,\theta,t)\ \mathbf{v}(r,\theta,t)\ ds\ dt. \tag{37}$$

where velocity $v(r,\theta,t)$ can be derived from the wave equation. The wave equation in a medium is defined as,

$$\nabla^2 \Phi = (1/c^2)(\partial^2 \Phi / \partial t^2) \tag{38}$$

where Φ is the velocity potential of the sound wave and ∇ is the Laplacian operator and c is the sound speed. The pressure in the compressive fluid resisting the motion of the sphere is defined by

$$P = \rho_0 (\partial \Phi / \partial t) \tag{39}$$

Solving the wave equation, Eq.(38), we can get the value of velocity potential as form of

$$\mathbf{v}(r,\theta,t) = (V_0 a_r^{\ 3}\cos\theta/2r^2)e^{-(c/a_r)t}$$
$$* \ [((2/a_r)-(r/a_r^{\ 2})-(1/r))\sin(c/a_r)t'$$
$$+ \ ((r/a_r^{\ 2})-(1/r)\cos(c/a_r)t']$$
$$+ \ V_0 a_r^{\ 3}/r^3\cos\theta \qquad (40)$$

and pressure is

$$P(r,\theta,t) = \rho_0 c V_0 a_r^{\ 2}/r^2\cos\theta[(1-r/a_r)\sin(c/a_r)t'$$
$$+ \ (r/a_r)\cos(c/a_r)t']e^{-(c/a_r)t'} \qquad (41)$$

Substituting Eq. (40) and (41) into Eq. (37), we obtain

$$E(r) = 1/2\ m'V_0^2(1 + a_r^{\ 3}/r^3) \qquad (42)$$

When $a_r = r$ is on the surface of the vibrating object, $E(r)$ equals the work done by the source, that is

$$E(r) = m'V_0^2 \qquad (43)$$

This equation is equal to the work done by the sphere on the medium by impulsive acceleration and is strongly similar to the equation (34) of impact energy loss which transferred to lateral vibration and radiated acoustic noise.

3. Acoustic Pressure Field of Two Sphere Impacts

In a sphere impact case, the acoustic radiation that comes from each sphere is equal to the sum of those independent sources at any point in the field (Koss and Alfredson, 1972). We assume the reflection effects between two spheres are not influenced in our analysis.

According to the Eq. (38), the wave equation, we can

translate it to express in the spherical coordinate system. and Figure 7 shows a typically spherical coordinates.

$$\nabla^2\Phi(r,\theta,t) = (1/c^2)(\partial^2\Phi(r,\theta,t)/\partial t^2) \qquad (44)$$

where ∇, Laplacian operator, equal to

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2 \qquad (45)$$

and c, sound speed, equal to $j\nabla p/\omega\rho$, in the air with ρ, the air density and $j = \sqrt{-1}$.

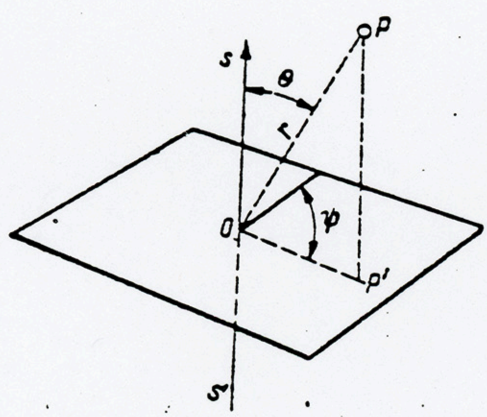


Figure 7: The typically spherical coordinates.

The acoustic pressure p and radial particle velocity u_r are given as, (Burnett, 1987:688),

$$p = \rho(\partial\Phi/\partial t) = \rho(\partial\Phi(r,\theta,t)/\partial t) \qquad (46)$$

and

$$u_r = -\nabla\Phi = -(\partial\Phi(r,\theta,t)/\partial r) \tag{47}$$

where ρ is the ambient air density and r is the radius to any point in the sound field, which surrounds the impact spheres.

From an oscillating spherical source system the velocity potential of the wave as given in (Malecki, 1969:175-176), is

$$\Phi(r,\theta,t) = \frac{a_r^3 v(1 + jkr)\cos\theta \; e^{j(\omega t - k(r-a_r))}}{r^2 \; [2(1 + jka_r) - k^2 a_r^2]} \tag{48}$$

where a_r is the radius of impulsive sphere, c is the sound speed in a sound medium, v is the velocity amplitude, ω is the angular frequency of vibration, k is the wave number and equal to ω/c. By Fourier synthesis, the arbitrary velocity potential can be obtained from Fourier's transform of the arbitrary velocity, $v(\omega)$. Thus, the velocity potential for an arbitrary velocity $v(\omega)$ is

$$\Phi(r,\theta,t) = \frac{a_r^3\cos\theta}{2\pi r^2}\int_{-\infty}^{\infty} \frac{v(\omega)(1 + jkr)e^{j[\omega t - k(r-a_r)]}d\omega}{2(1 + jka_r) - k^2 a_r^2} \tag{49}$$

To solve the velocity potential produced by the Hertzian's sphere acceleration, the convolution equation is useful. Koss and Alfredson developed the equation for convoluting the forcing function and impulse solution and is given by

$$\Phi(t') = \int_0^T I(t' - \tau)\, F(\tau)\, d\tau \tag{50}$$

Substituting equation (49) and (33) into (50), we obtain

$$\Phi(r,\theta,t) = \frac{a_m\ a_r^{\,3}\ \cos\theta}{2r^2}\int_0^T(1+e^{-g(t'-\tau)}$$

$$\times\ [(2r-a_r/a_r)\sin g(t'-\tau)\ -\ ((\cos g(t'-\tau)\ +\ 1)]$$

$$\times\ \sin g\tau\ d\tau \tag{51}$$

Because the acoustic pressure is given by

$$p = \rho(\partial\Phi/\partial t), \tag{52}$$

we can get the peak amplitude of acoustic pressure at near field and far field by the general equation derived by Koss and Alfredson for a single impacting sphere of radius a_r, as

$$p(r,\emptyset,t)= \frac{\rho a_m a_r^{\,3}\cos\theta}{4(\omega^4+4g^4)2r^2}\ \{[(2r/a_r)-1][(8g^3\omega-4g\omega^3)\cos\omega t'$$

$$+\ 8\omega^2g^2\sin\omega t']\ -\ 4\omega^4\sin\omega t'\ -\ (8g^3\omega\ +\ 4\omega^3g)\cos\omega t'$$

$$+\ (2r/a_r\ -\ 1)[(4\omega^3g\ -\ 8\omega g^3)\cos gt'-\ (8\omega g^3\ +\ 4\omega g)$$

$$\times\ \sin gt']e^{-gt'}$$

$$+\ [(4\omega^3g\ -\ 8\omega g^3)\cos g(t'-\ \pi/2g)\ -\ (8\omega^3g+\ 4\omega^3g)$$

$$\times\ \sin g(t'\ -\ \pi/2g)]e^{-t'g}\}$$

$$+\ (\rho A a_r^{\,3}\cos\theta\sin\omega t'/2r^2) \tag{53}$$

where $\omega = \pi/T$, $g = c/a_r$, a_m is the acceleration peak amplitude, r is the radial distance from the impact point, $t' = t-(r-a_r)/c$ is the time difference of sound propagation of two impact bodies, and θ is the azimuth angle of the measurer. The acoustic pressure field generated by two object impacts is the sum of the acoustic pressure by each object (Koss and Alfredson). The time difference of sound propagation is existed, if the observer's position is not along the line of impact. The time delay, $t'= t - (r-a_r)/c$,

is equal to

$$t' = |r_2 - r_1| \div c \tag{54}$$

4. Acoustic Pressure Field of Meshing Gear Impact

The contact problem at the points where the teeth touch each other is found by means of the Hertzian contact theory (Colbourne, 1987:244). Since the tooth impact takes place along a line which perpendicular to the involute gear profile of both teeth, the impact direct noise of the meshing gear tooth impact may be represented by two spheres' Hertzian impacting sound field. The radii of contact point of each tooth are equal to the radius of two spheres.

Because the impact on the meshing gear teeth was taking place suddenly, the amount of energy which partitioned into the natural mode of vibration of the impact spheres is small in comparison with the original kinetic energy of the impact spheres (Rayleigh). At the same time, the duration which remains contact of impact is very long compared with the lowest mode of vibration of spheres (Koss, & Alfredson, 1974). Thus, the effects of vibration within the impact bodies on the sound field can be neglected in the analysis. In the case of meshing gear teeth, it means that impact direct noise generated by tooth impact won't be influenced by lateral vibration which transferred through the shaft and bearing to gear box surface and radiated.

Because acoustic field theory is concerned with the

description of the pressure field produced by real objects in an acoustic medium, thus, the acoustic field radiated by a sphere undergoing a Hertzian acceleration can be obtained from the Kirchhoff's radiation field that generated by an impulsively transnational acceleration of a sphere.

In the case of meshing gear tooth impact, the radii of meshing gear teeth at the contact position are always different except for the contact position at pitch circle. The acoustic pressure amplitude, $p(r,\theta,t)$ of meshing gear tooth impact at any point in field was given by the sum of the sound pressure radiated by the meshing gear teeth, respectively. The form was given by Koss and Alfredson, as

$$p(r,\theta,t) = p(r_1,\theta_1,t) + p(r_2,\theta_2,t) \tag{55}$$

Because the theory derived by Koss and Alfredson is based on two sphere impacts, the profile of gear teeth are not real sphere. At the same time we are only interested in the peak acoustic pressure amplitude produced by the meshing gear impact, the angle of observer, θ, will always equal to zero. To measure the acoustic pressure of meshing impact direct noise, therefore, the equation (54) has this requirement to make some modification to adapt for the specific situation of meshing gear impact. Referring to the study of Richards, Westcott and Jeyapalan, the sound field pressure of the meshing gear impact will become for peak pressure amplitude, $\theta = 0$, as

$$P_0\ (r,t) = [\rho a_r c v_0 / 2(4z^4 + 1)r]$$
$$\times \{(2z^2 - 1)\cos(\pi t'/T) + 2z\sin(\pi t'/r_0)$$
$$+ [(1 - 2z^2)\cos(ct'/T) - (2z^2 + 1)$$
$$\times \sin(ct'/a_r)]e^{-ct'/a_r}\} \qquad (56)$$

where z is non-dimension duration, $z = cT/\pi a_r = g/\omega$, and $a_m = v_0\pi/2T$. If the parameters are replaced by

$$\tau = t'/T, \qquad \Gamma = \sin^{-1}(2z/\sqrt{(4z^4+1)}), \text{ and}$$
$$\Omega = \sin^{-1}[(2z^2 + 1)/\sqrt{2}\sqrt{(4z^4 + 1)}]$$

, the expression of P_0 can be expressed more simply as

$$P_0 = (\rho a_r c v_0 / 2r\sqrt{(4z^4 + 1)}$$
$$\times [\cos(\pi\tau - \Gamma) + \sqrt{2}(e^{-\pi z\tau}\cos(\pi z\tau + \Omega))] \qquad (57)$$

For the far field pressure P_0, after $t' = T$, the expression can be expressed, for $\tau = t'/T > 1$, as

$$P_0 = \rho c a_r v_0 / 2r\sqrt{(4z^4 + 1)}$$
$$\times \{e^{-\pi z(\tau-1)}\cos[\pi z(\tau - 1) + \Omega]$$
$$+ e^{-\pi z\tau}\cos(\pi z\tau + \Omega)\} \qquad (58)$$

For the shape difference with sphere, the equation (58) will be modified, by Richards, Westcott and Jeyapalan, as

$$P_{0max} = \rho c a_r v_0 / 2r\sqrt{(4z^4 + 1)}$$
$$\times [\cos\{(2.3/z)^{1/3} - \Gamma\}$$
$$+ \sqrt{2}(e^{-(1.5z)^{2/3}}\cos\{(1.5z)^{2/3} + \Omega \qquad (59)$$

Thus, the acoustic pressure of meshing gear impact or the others that are not too different shaped bodies impact can be calculated by this equation at a distance, r.

NUMERICAL IMPLEMENTATION AND ANALYSIS

With the advent of high speed digital computer and the development of finite element analysis software, the structure modal analysis is usually obtained. The result of these operations give the desired modes in terms of their frequencies and geometries. The impact noise influence as well as the continuous noise level produced by the vibration of machine structure is generally unacceptably high such as in a forging machine and drop hammer. The meshing gear impact noises might be produced same unacceptable noise.

A typical noise signature of a meshing gear impact may be divided into two main categories: gear whine and gear rattle noise (Blankenship and Singh, 1992). It is same classification with the results of J. D. Smith and mentioned in the previous paragraph. Gear rattle noise, the main study in this research, is a highly nonlinear, and impulsive phenomenon which generally occurs under lightly loaded conditions and consists of repeated gear tooth impacts through backlash due to torsional vibrations of the geared system (Johnson and Hirami, 1991).

In order to achieve effective noise control in a gear system, a technique for predicting the direct response of meshing gear tooth impact resulting from backlash tolerance is required. The backlash may be caused by the tolerance of installation, manufacturing, or operation temperature increases. The prediction procedures of the meshing gear

impact direct noise will follow three stage: (1) finite element analysis of the meshing gear impact mode shape and frequencies of the vibration, (2) numerical calculation of the impact response of acoustic pressure amplitude for each mode, and (3) prediction of sound radiation from the above data and the data of different backlash and its resulting impact load.

Finite Element Frequency and Modes Analysis

The Finite Element method used in this research is to predict the natural frequencies and elastic mode shape analysis of the meshing gear impact. Some samples of the modes predicted are shown. In a case of meshing gear tooth impact we can depict their motion by modal of block and spring (Spoyts, 1984). Where the stiffness of tooth pair is expressed as spring constant. Therefore the impact of meshing gear teeth can be shown as Figure 8. The motion of this meshing gear impact described by Lagrange's equation is, (refers to Fig. 8)

$$L = 1/2\ (m_1\dot{x}_1^{\,2} + m_2\dot{x}_2^{\,2}) - 1/2*k(x_2-x_1) \qquad (60)$$

Then the equations of motion are

$$m_1\ddot{x}_1^{\,2}-k(x_2 - x_1) = F_0\delta(t)$$

$$m_2\ddot{x}_2^{\,2} + kx_2 = 0 \qquad (61)$$

In matrix form this becomes

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1^{\,2} \\ \ddot{x}_2^{\,2} \end{Bmatrix} + \begin{bmatrix} k & -k \\ 0 & k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_0\delta(t) \\ 0 \end{Bmatrix} \qquad (62)$$

$$[M]\{\ddot{x}\} + [K]\{x\} = F_0\delta(t) \tag{63}$$

If $\omega = \sqrt{(k/m)}$ and $x_i = u_i\cos(\omega t + \emptyset)$, the equation can be expressed by the form of

$$-\omega^2[M]\{\ddot{u}\} + [K]\{u\} = F_0\delta(t) \tag{64}$$

When the mode and frequency problem need to be solved, the right hand side is equal to zero.

$$-\omega^2[M]\{\ddot{u}\} + [K]\{u\} = 0 \tag{64a}$$

It is the form of familiar eigenvalue problem that can use for solving the meshing gear teeth's modes and frequencies.

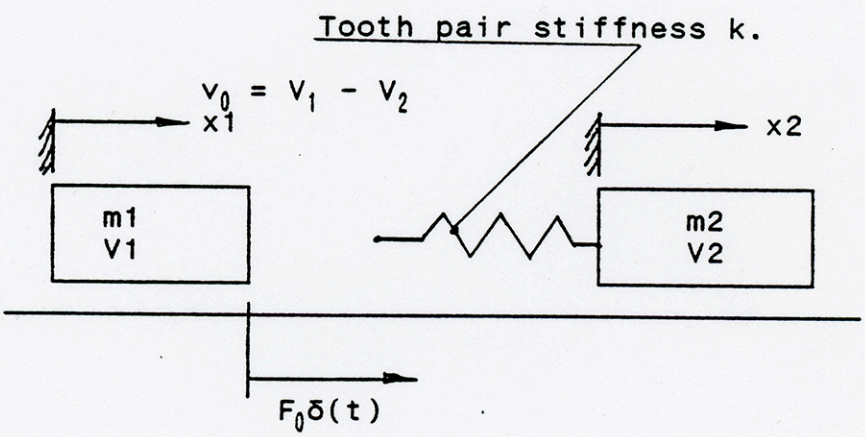


Figure 8: The impact applied on meshing gear model.

Because the impact of meshing gear teeth is at the side of the tooth profile, the transient vibration is mainly excited on the teeth. Therefore the full one tooth is only needed for finite element modeling, Figure 9. The finite gear tooth element model of this research is represented by 30

nodes and 20 quadrilateral plate elements. The natural frequencies and shapes were predicted by the finite element computer software package IMAGES-3D. Modes up to 10 were auto calculated. The mass matrix is generated by lumped weights without rotary weight technique. A sample of the modes predicted was shown on Figure 10. In general the low frequencies are dominated by the result of the heavy mass of the impactor (Lam and Hodgson,1993). The gear teeth's mass is relatively small compared to whole machine and the thickness of tooth is relatively long compared with wave length of sound propagation, The natural frequencies and modes found by finite element are very high in frequencies, the first modes was shown at frequency of 5.852 kHz and the second mode was found at frequency of 7.898 kHz. It is matched very well with the study of Lamb (Koss & Alfredso, 1974) that the natural frequencies of vibration of spheres are ultrasonic. According to the study of Rayleigh (Koss & Alfredson, 1974) the effects of modal vibrations within the sphere on the sound field can be neglected in the analysis. Therefore to understand and predict the impact direct noise of the meshing gear, the numerical implementation will calculate the frequency range from 500 Hz to 3000 Hz. Because the human ear is also most sensitive to frequencies above 500 Hz (Skaistis, 1987).

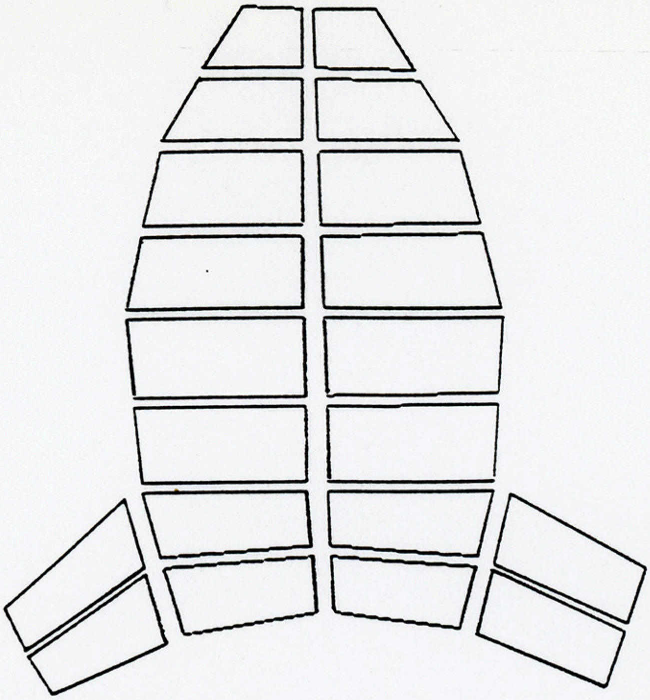


Figure 9: The finite element mode of gear tooth.

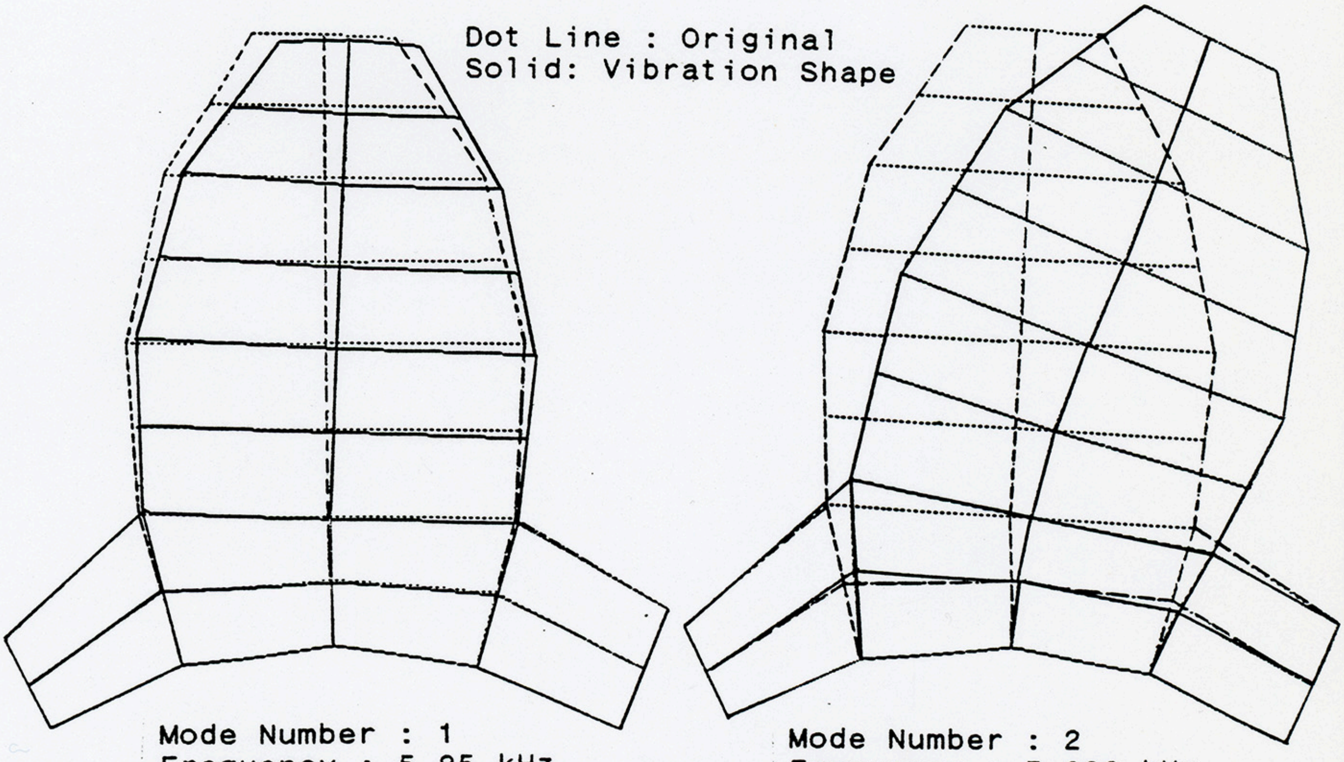


Figure 10: The sample modes and frequencies of tooth.

Numerical Implementation

For a system with many parameters and degrees of freedom solutions,the acoustic pressure radiation is obtainable only through numerical technique. Because the Helmohltz's sound field can only deal with one frequency at one time, the noise prediction of meshing gear impact will choose 4 different frequencies, 500, 800, 1800, and 3000 Hz etc. The various frequencies can get from the assumption and finite element method and the other parameters will be defined from the fundamentals of a gear system. The procedure of numerical computation is shown by the flow diagram of Figure 11.

The parameters of spur gear is given as:

n : is the number of rotation, 800 rpm.

P_d: is the pitch diameter, 11.34 in.

N_t: is the number of teeth, 32T.

D_0: is the outside diameter, 11.53 in.

W_f: is the face width, 1.5 in.

C_d: is the center distance, 7.15 in.

D_b: is the base circle diameter, 10.66 in.

P_w: is the power input for transmission, 10 Hp.

The data of backlash should be chosen to meet the requirements of the application. In power gearing, the amount of backlash should be at least enough to let the gears turn freely. The suggested backlash is shown at Table I (Dudley, 1984:3-20).

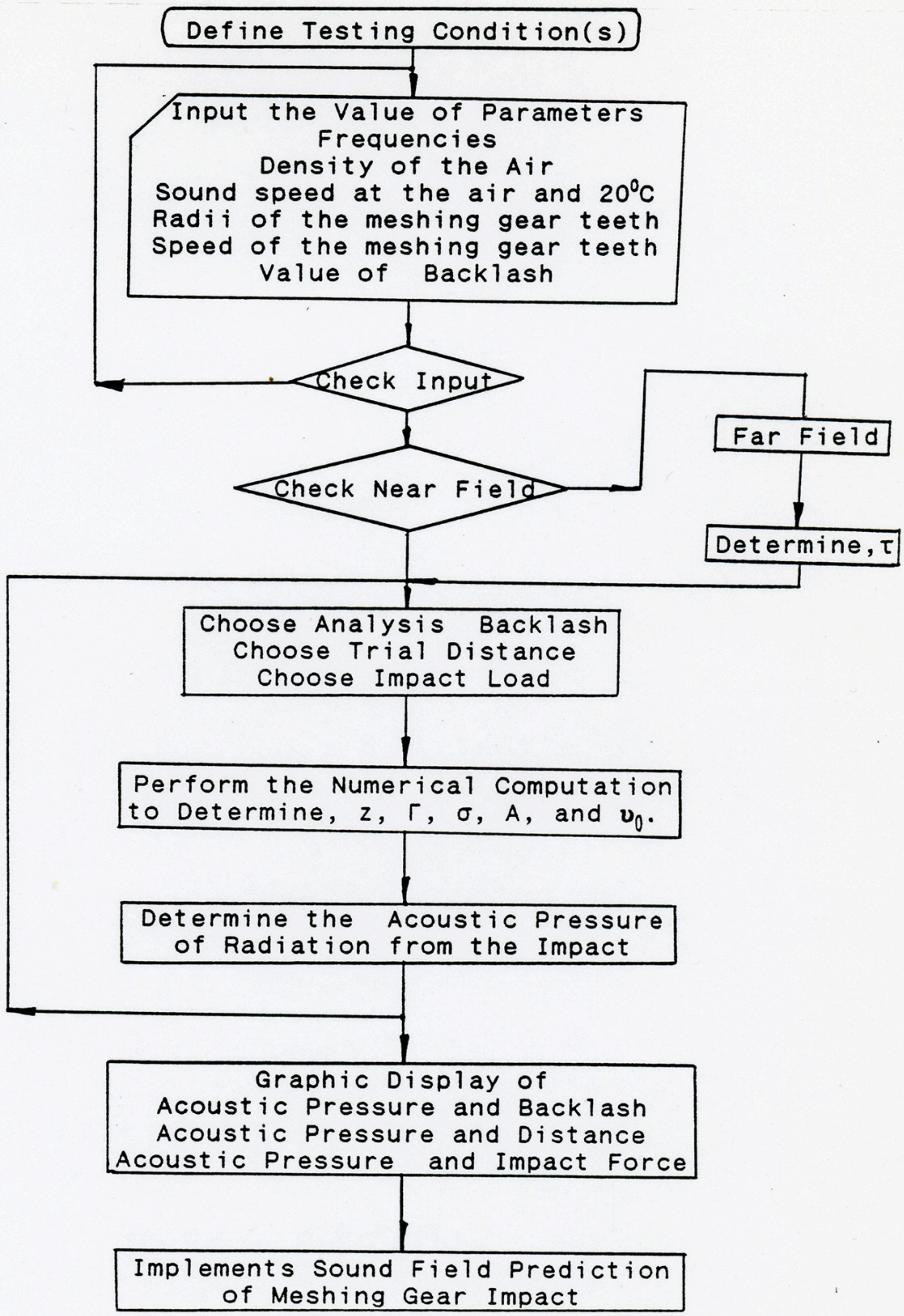


Figure 11. The flow diagram of numerical implementation.

Table I: Suggested backlash when assembled.

Metric		English	
Module	Backlash, mm	Diametral pitch	Backlash, inches
25	0.63-1.02	1	0.025-0.04
18	0.46-0.69	1.5	0.018-0.027
12	0.35-0.51	2	0.014-0.020
10	0.28-0.41	2.5	0.011-0.016
8	0.23-0.36	3	0.009-0.014
6	0.18-0.28	4	0.007-0.011
5	0.15-0.23	5	0.006-0.009
4	0.13-0.20	6	0.005-0.008
3	0.1-0.15	8 and 9	0.004-0.006
2	0.08-0.13	10-13	0.003-0.005
1	0.05-0.10	14-32	0.002-0.004

Meshing Gear Impact Sound Field Prediction

From the Eq. (59) the sound field pressure of meshing gear impact can be computed for the body of radius a_r at the distance r. The total sound field pressure of the impact bodies is the summation of both pressure to measurer. In the case of meshing gear impact the both radii at the position of impact gear teeth are required for calculation. According to the geometry of involute spur gear, the first impact position of meshing gear teeth can be calculated by the formula (Dudley, 1987:2-30) of

$$\rho_1 = \sqrt{(r_0^{\ 2} - (p_d \cos\varnothing)^2)} - p\cos\varnothing.$$

$$\rho_2 = C_d \sin\varnothing - \rho_1. \qquad (65)$$

where ρ_1: is the tooth #1 first contact radius.

ρ_2: is the tooth #2 last contact radius.

C_d: is the center distance of the meshing gear.

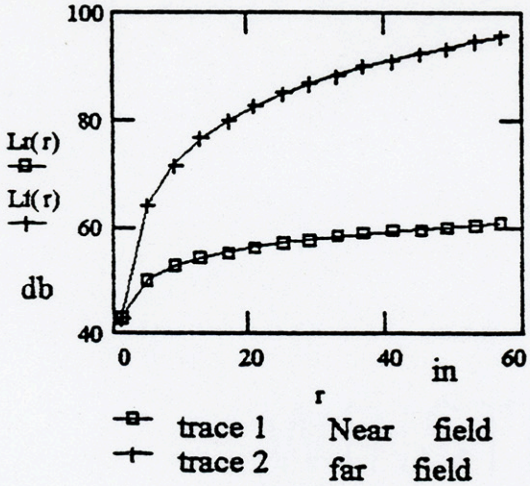
p_d: is the pitch diameter of the gear #1.

p : is the circular pitch.

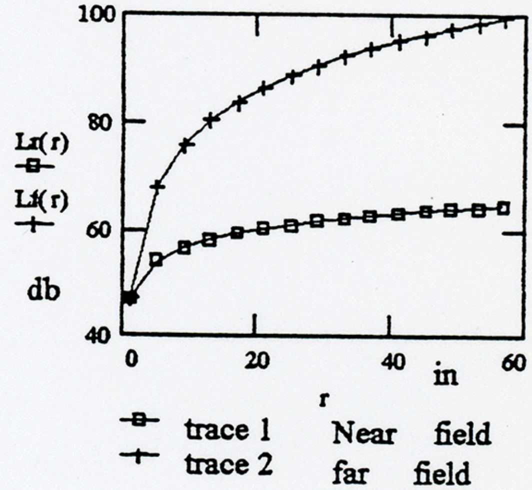
$\varnothing$: is the pressure angle.

The value of ρ_i is the parameters for acoustic pressure computation of meshing gear teeth. The relationship of total peak noise pressure level and the measuring distance was shown at Figure 12, in which the relationship of sound pressure level against measuring distance from the contact position is almost same while frequency under 1 kHz. The sound pressure level difference of near field and far field is about 25-30 decibels, because the sound pressure level of noise of near field is only considered the one that closed to the measurer. Figure 13 is the related chart of peak pressure level and the backlash. The relationship of peak pressure level and impact loads are shown at Figure 14. Those charts were shown that the backlash and impact load against sound pressure level are same, because the backlash is the important component of impact load of meshing gear. In practical the sound pressure level of noise with unit decibel is more useful than acoustic pressure amplitude with the unit of bar or psi. The peak pressure can be translated to the form of sound pressure level by the equation (Busch-Vishniac & Lyon, 1981) of

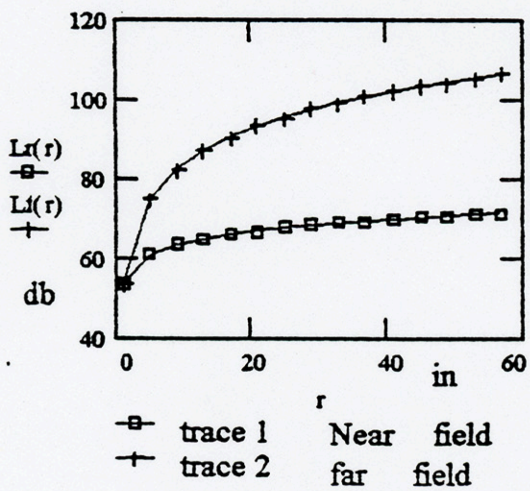
$$L = 10*\log[(4\pi r^2/\rho_0 c)(\int Pmax\ dt)] \qquad (66)$$



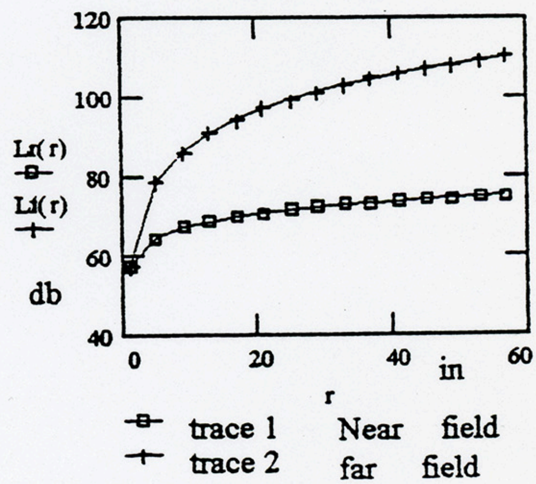
(a) Frequency ω is 500 Hz.



(b) Frequency ω is 800 Hz.

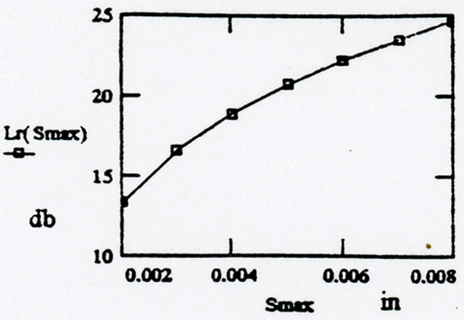


(c) Frequency ω is 1800 Hz.

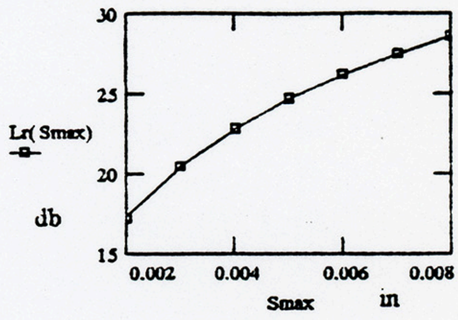


(d) Frequency ω is 3k Hz.

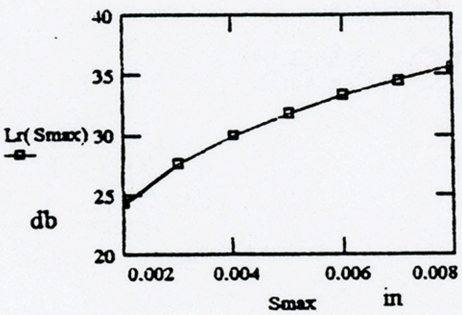
Figure 12: The amplitude of sound pressure against measuring distance.



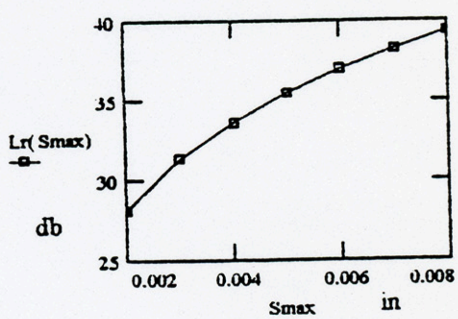
(a) Frequency ω is 500 Hz.



(b) Frequency ω is 800 Hz.

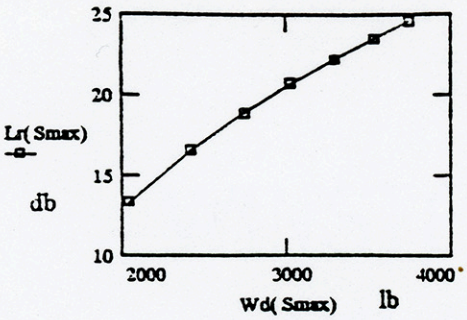


(c) Frequency ω is 1800 Hz.

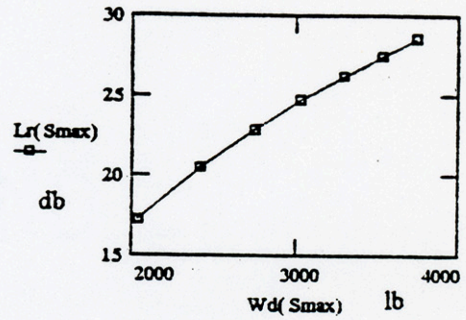


(d) Frequency ω is 3k Hz.

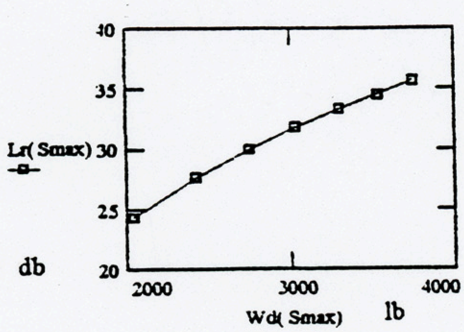
Figure 13: The amplitude of sound pressure against backlash.



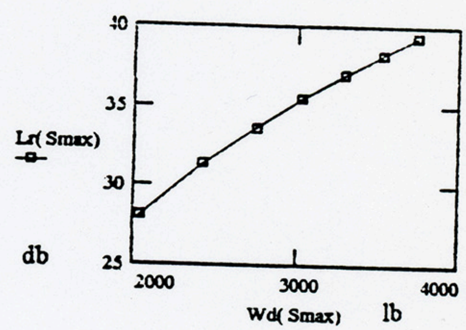
(a) Frequency ω is 500 Hz.



(b) Frequency ω is 800 Hz.



(c) Frequency ω is 1800 Hz.



(d) Frequency ω is 3k Hz.

Figure 14: The amplitude of sound pressure against impact loads.

To verify the prediction technique that presented in this study, the comparison of different methods is required. Refers to Appendix, acceleration noise calculation is an empirical method and used for machine impact noise estimation. Because the frequency of acceleration noise is estimated about 750 Hz of sphere acceleration, the verification only compares with the noise of 500 and 800 Hz and shows as Table II. There are fitted very well only about 3.4 db difference.

Table II: Comparison of different predicted methods.

Average sound pressure level (db)		
Center frequency	Predicted	Acceleration method
500 Hz.	86.1	81.7
800 Hz.	89.3	85.2

CHAPTER V

SUMMARY

In a gear system, its noise can be divided into two categories: gear whine and gear rattle. Gear rattle is a highly nonlinear, impulsive phenomenon that generated by the condition of light load and tooth impacts through backlash due to torsional vibration (Blankenship, & Singh, 1992:81). The gear rattle noise is the main concern of this study and develops a technique to predict it. Both the gear whine and gear rattle are basically due to the dynamic load of the meshing gear teeth (Lin, 1987). Reducing the gear dynamic load or impact will reduce the noise of a gear system.

There are two loads happened during the process of gear tooth engagement, which are the acceleration load and impact load or dynamic load (Buckingham, 1949:427). The impact load or dynamic load is mainly influenced by the transmission error, in which the transmission error is defined as the difference of output shaft position between the actual shaft and the perfect output shaft. The transmission error is consisted by backlash of assembly, manufacturing error, deformation of the load, and wear. The caused reasons of dynamic load of meshing gear have divided into three sources, by Dr. Kubo, (1) the fluctuations of input and output torque caused by more or less rough running of the engine, (2) the fluctuation of torque resulting from torsional vibration of

the power transmission system, and (3) fluctuation of torque as teeth roll through mesh. The is caused by period changes in tooth meshing stiffness and by transmission error. Because the backlash is the medium of gear rattle due to torsional vibration, the backlash is more concern in this study.

The impact action will happen when the meshing gear teeth lose their contact which is caused by light input load and high inertia of a gear system. The impact action of a meshing gear will transfer most of the impact kinetic energy to elastic strain energy, but a portion is dissipated and transfer to plastic strain energy, fracture, light, and sound etc. (Brach, 1991:35). It is called energy 'loss' of impact. In the case of meshing gear tooth impact, the energy loss is studied by Smith and equal to 8/9 of input kinetic energy, when the impact velocity is faster than 0.2 m/sec. These dissipated kinetic energy of a pair of meshing gear teeth is the force source of the gear whine and gear rattle noise. The portion of gear rattle energy is equal to the work done on the fluid medium, which surrounds the object, by the impulsively accelerated object.

The impact of meshing gear teeth can be depicted by the motion of two sphere impacts, because the thickness of a tooth is too long compared with the wave length of impact vibration propagation and the mass of the tooth is too small to excite the damping effect while impact. Therefore the rattle noise, impact direct noise, can be calculated by the equation which

developed from the two sphere impacts. To more precisely describe the impact of meshing gear teeth, the form of two sphere impact is modified by Richardsons for impact direct noise prediction, as

$$Pmax := \rho \cdot c \cdot a_r \cdot \frac{v_0}{2 \cdot r \cdot \sqrt{4 \cdot z^4 + 1}} \cdot \left[\cos\left[\left(\frac{2.3}{z} \right)^{\frac{2}{3}} - \Gamma \right] + \sqrt{2} \cdot \left[e^{-(1.5 \cdot z)^{\frac{2}{3}}} \cdot \cos\left[(1.5 \cdot z)^{\frac{2}{3}} + \Omega \right] \right] \right]$$

Thus the acoustic pressure of meshing gear rattle noise can be calculated by this equation at a distance, r.

The finite element analysis is a nice tool to predict the modes and frequencies of the impact vibration of a structure. The modes and natural frequencies of the meshing gear impact are calculated by the FEA software package IMAGES-3D. The gear tooth element model is represented by 30 nodes and 20 quadrilateral plate elements. Because the low frequency is dominated by the mass of impactor, the natural frequencies of meshing gear tooth impact tends to high frequencies. The first mode and natural frequency are found at the frequency 5.852 kHz. It is match well that the study result of Koss and Alfredson. To predict and understand the impact direct noise of meshing gear teeth, the prediction technique are applied for the frequency range from 500 Hz to 3 kHz. due to the human ear sensitive to the frequencies higher than 500 Hz.
The prediction was made between the sound pressure level against measuring distance, backlash and impact load and shown as Fig. 12-14.

CHAPTER VI

CONCLUSIONS

An analysis and a finite element model were developed to investigate the impact effect response of rattle noise on the surface of meshing gear teeth. The impact direct noise of meshing gear teeth was predicted by the technique developed in this study based on the data found from the finite analysis. Applying this prediction technique to a pair of identical spur gears revealed the following conclusions.

The hypothesis that impact direct noises are significant in overall gear system's noise excitation has been proved. The sound pressure level of impact direct noise can reach as high as 90 db, when the frequency is 800 Hz, measuring distance, r, is 20 inches, and the backlash is 0.004 inches. The backlash has the effect that not only influences the structure vibration noise, studied by previous researchers, but also dominates the impact direct noise of a gear system. The backlash will significantly contribute impact direct noise , but the magnitude of impact direct noise which produced by the backlash is much lower than those of overall gear system's noise. When the noises are predicted at backlash 0.004 inches, frequency 500 Hz, and measuring distance 20 inches, the overall system's noise is 84 db higher than that of impact direct noise caused by backlash, 17.5 db. The range of noise contribution is also limited in which the total

sound pressure level only has 14-24.5 db.

The difference of frequency has inverse ratio, when the sound pressure level is predicted at backlash 0.004 inches, measuring distance 20 inches, and the frequency range from 100 Hz to 3000 Hz. It is implied that the frequency is limited influence to sound pressure level excitation of meshing gear tooth impact. This relationship also proved at the chart of sound pressure level against measuring distance, where the sound pressure levels of prediction are almost same, if the frequency below or over 1 kHz. Because it is the frequency to classify near field and far field.

The impact load is also a significant factor that influence the impact direct noise of meshing gear system, but the range of variation and value are limited and similar to those contribution of backlash. Because the backlash is the key factor of the impact load excitation of a meshing gear teeth. The impact load and backlash have almost direct relationship with the sound pressure level of impact direct noise.

Because the natural frequencies of meshing gear belong to high frequency range, which are limited influence to human ear, the method of modes and frequencies prediction by finite element technique is not a best way for impact direct noise prediction. However, the results obtained herein should be useful for predicting the impact direct noise excitation of spur gear system, and for modifying tooth backlash and impact

load dynamic performance. To fully understand the impact direct noise of a spur gear system, it is recommended that experimental tests should be performed to verify the prediction technique developed in this study.

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APPENDIX

ACCELERATION NOISE

When the machine has impact motion, the non-resonant modes of the machine also have contribution to some of the smooth noise background. But their vibrational energies were very low and their mode shapes didn't exist tremendously high radiation efficiency. A more likely noise source is the acceleration noise.

The total amount of sound energy radiated by the impact bodies due to acceleration noise can be estimated by the empirical formula (Richards, Westcott, & Jeyapalan, 1979), as

$$E_r = [(1/2)*\rho V u_0^2]*g(\delta)$$

$$\delta = ct/V^{1/3}$$

where $g(\delta) = 0.7$, for $\delta<=1$

$= 0.7*\delta^{-3.2}$, for $\delta>1$

V : is the volume of the impactor.

t : is the duration time of impact.

u_0: is the velocity amplitude of the impactor prior to impact.

The frequency of the acceleration noise peak can be estimated from empirical equation (Lam, & Hodgson, 1993) for the case of two sphere impacts, as

$$f = 76.1/(a_r/2)$$