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MATHEMATICAL OPERATIONS USED IN ORDINARY OCCUPATIONS
AND SUGGESTIONS FOR INCORPORATING THEM IN
HIGH-SCHOOL MATHEMATICS

A Thesis Submitted to the Graduate Division in
Partial Fulfillment of the Requirements for the Degree
of Master of Science

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By

W. R. Binns

KANSAS STATE TEACHERS COLLEGE

Pittsburg, Kansas

August, 1938

WITHDRAWN

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To those marking the check list the writer is indebted for the data used in this study.

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CHAPTER I

INTRODUCTION

Purpose of the Study

The purpose of this study is to determine the type problems in algebra, geometry, and general mathematics that are most frequently used by high school graduates. The applicability of these mathematical operations to ordinary problems of daily life is to be decided by means of a survey among high school graduates who have had courses in algebra and geometry. An enumeration and analysis of the type problems that are most used by these graduates will give a much better conception of what should be taught in a general mathematics course. This investigation proposes to apply the criterion of usability in the determination of what type problems are best suited to a general mathematics course for secondary schools.

Philosophy upon Which this Study is Based

Education must provide the child with experiences which not only seem worth while to him but also have value in helping him adapt himself to present and future life situations. John Dewey expresses this educational philosophy as follows:

A society marked off in classes need be especially

sensitive only to the education of its ruling elements. A society which is mobile, which is full of channels for the distribution of a change occurring anywhere, must see to it that its members are educated to personal initiative and adaptability. Otherwise, they will be overwhelmed by the change in which they are caught and whose significance or connections they do not perceive.¹

Mathematics, to fit into the present system of mass education, must contain that type of material which the masses actually use. The mathematician contends that,

A theorem of geometry is useful and practical if it arouses one's curiosity and gives him an hour of pleasant thinking, quite as well as if it helps to lay out the foundation of a house. However, this type of objective must supplement rather than supplant the more basic instruction which must keep close to earth if mathematics is made a subject of general instruction.²

In the contemporary secondary school, it no longer seems

. . . probable that mathematics can continue to justify itself on the basis of classical tradition, for we cannot appeal to the mystical influence of mathematics to develop powers of memory, imagination, or judgment.³

If mathematics is to maintain a place of importance in

¹As quoted in William H. Kilpatrick, Source Book in the Philosophy of Education (New York: The Macmillan Company, 1924), p. 137.

²K. P. Williams, "Why We Teach Mathematics," Mathematics Teacher, XXIX (October, 1936), 279.

³A. C. Rosander, "Quantitative Thinking on the Secondary School Level," Mathematics Teacher, XXIX (February, 1936), 61.

the high-school curriculum, it must be practical for a greater number of students. Quite in keeping with a utilitarian philosophy of education, this study attempts to determine the usable material available in present mathematics textbooks.

Need for the Study

The preceding quotations show that formal mathematics, as it has been taught in the high schools, is being severely criticized from the standpoint of usability in life situations. General mathematics has been placed in the curriculum for many of the students who do not wish to specialize in mathematical fields. There are now two types of general mathematics courses offered. One of these contains a sampling of advanced arithmetic, algebra, and geometry without regard to usability; the other is composed of problems most used in life outside of school. Courses of the former type have greater apparent value for the student who goes on to college and is engaged in technical fields, whereas the latter type courses are designed for the majority of secondary-school students who will not choose a technical type of work as a vocation. It is the second type of general mathematics courses which are to be considered in this study.

The teaching of mathematics has reached a crisis because some school people have insisted that the high-school student,

"Take algebra and geometry or nothing."⁴ This crisis may be averted by providing mathematical courses which give due consideration to the needs, interests, and abilities of the majority of ninth-grade students. The limitations of the transfer of training concept have been realized during the past few years. This realization has led many educators to feel that a large part of the content of algebra and geometry textbooks has little practical value for most students after they leave high school. For this reason general mathematics has been introduced into the curriculum of many high schools.

Since general mathematics has been adopted to fill a gap unprovided for in traditional mathematics courses, it is necessary that the content of these general mathematics courses embrace the most usable and practical mathematics. A determination of this "most usable content" is urgently needed and the recommendation of suitable subject matter will be equally valuable to curriculum maker and instructor.

Similar Studies

In order to find the mathematics which should enter into the curriculum of general education, Dobbitt sampled the mathematics used in several scientific magazines. He found that more arithmetic should be taught. He also discovered

⁴Virgil S. Mallory, "Providing for Individual Needs in Mathematics," Mathematics Teacher, XXX (May, 1937), 214-220.

that the vocabulary of geometry was relied upon in the visualization of the forms of objects, and that a moderate amount of algebra was needed, especially in the interpretation of graphical comparisons.⁵

Kilzer and Thompson, who made a study of general mathematics in Wyoming during 1934, found that (1) the number of Wyoming high schools offering general mathematics courses has nearly doubled in recent years, (2) textbooks in general mathematics give more attention to algebra than they do to arithmetic or geometry, and (3) about one-half of the universities in the United States are accepting high school credit in general mathematics in lieu of algebra.⁶

Thorndike, in his study of the use of algebra in the scientific fields, points out that many things taught in the regular algebra course are of limited use. He also states that a few other processes, especially graphs, which are not especially emphasized in the classroom prove highly useful in handling problems connected with the interpretation of scientific data.⁷

⁵Franklin Bobbitt, Curriculum Investigations (Chicago: The University of Chicago Press, 1926), pp. 119-150.

⁶L. R. Kilzer and Charles H. Thompson, "Status of Algebra and General Mathematics in Ninth Grade," School Review, XLIII (June, 1935), 446-450.

⁷Edward L. Thorndike, The Psychology of Algebra (New York: The Macmillan Company, 1923), pp. 47-93.

Restatement of the Problem

The problem under consideration is a determination of the type problems most needed by people in the common occupations represented in Kansas communities so that these problems might be incorporated into a general mathematics course.

There is disagreement among educators concerning the content of the general mathematics textbook. Since the Kansas state department of education now endorses a general mathematics course in the high schools, the need for a textbook which contains the basic type problems which are most commonly used is evident. Hence a study to determine the problems most valuable from the utilitarian standpoint is needed. To meet this need the present study is undertaken.

CHAPTER II

COLLECTION AND COMPARISON OF DATA

Source and Method of Securing Data

The data for this study were secured by means of a specially constructed check list consisting of a sampling of type problems from the following four textbooks:

Englehardt and Haertter, First Course in Algebra, Philadelphia: John C. Winston and Co., 1927.

Strader and Rhoads, Plane Geometry, Philadelphia: John C. Winston and Co., 1927.

Schorling and Clark, Mathematics in Life, New York: World Book Company, 1937.

Lenner, Practical Mathematics, New York: The Macmillan Company, 1936.

Problems were selected with the idea of trying to sample fairly the three fields of arithmetic, algebra, and geometry. The final selection contained 137 problems, which were mimeographed to form the check list, a copy of which appears in the appendix of this study.¹ The three main divisions of the check list, namely, arithmetic, algebra, and geometry, were subdivided, and the problems grouped under the following headings:

¹The fact that the problems are numbered to 139 is a result of the unintentional omission of number 96 from the check list.

ARITHMETIC: Measurement, Fractions and Decimals, Percentage, Home Expenditures, Insurance, Areas, and Miscellaneous.

ALGEBRA: Graphs, Factoring, Fractions, Equations, Simple Processes, and Miscellaneous.

GEOMETRY: Construction, Triangles, Circles, Polygons, Parallels, Proportion, and Miscellaneous.

Copies of the check list were personally given out by the writer to 101 people representing thirty-eight different occupations. This group was composed of approximately as many women as men. All were high-school graduates who had completed regular courses in algebra and geometry. The occupations represented are listed in Table I, page 9. This distribution of occupations corresponds to that obtained by R. B. Russell in a random selection of the people from twenty-eight Kansas communities.²

The method of distribution, handing the check list to the persons for whom it was intended, provided an opportunity for the writer to explain the purpose of the survey and to give detailed instructions for filling out the check list. After a discussion of the check list a copy was left with each of the individuals from whom data for this study were secured to be filled out at his convenience. The filled-out check lists were picked up at a later date by the

²Raymond Bennett Russell, "General Business Needs in Agricultural Communities," (unpublished master's thesis, Kansas State Teachers College, Pittsburg, 1937), p. 12.

TABLE I

Occupations Represented by 101 People in Kansas Who Marked
Check Lists to Determine the Most Frequently Used Type
Problems in Algebra, Geometry, and General Mathematics

Occupation	Frequency
Farmer	41
Railroad Employee	4
Postal Employee	4
Miner	4
Groceryman	4
Housewife	4
Public and County Official	3
Mechanic	2
Dealer in Automobiles and Supplies	2
Barber	2
Filling Station Operator	2
Office Employee	2
Salesman	2
Teacher	2
Druggist	2
Janitor	2
Contractor	2
Real Estate and Insurance Salesman	1
Shoe Repairman	1
Hardware Dealer	1
Minister	1
Banker	1
Trucker	1
Plumber	1
Life Insurance Agent	1
Laundryman	1
Ice Cream Maker	1
Printer	1
Dentist	1
Physician	1
Veterinary	1
Lumberman	1
Auctioneer	1
Feed and Produce Salesman	1

writer.

Each of the 101 people, who checked the list of 137 mathematical processes, was asked to mark with a "2" those type problems he had solved often since leaving high school; to mark with a "1" those solved seldom; and to mark with a "0" those problems never solved.

Method of Tabulating and Comparing the Data

Table II, pages 11 to 13, shows how many of the 101 persons solved each problem "often", "seldom", or "never". Under the heading "often" is listed the number of persons who solved each problem many times. Under the heading "seldom" is the number of persons who solved each problem seldom. Under the heading "never" is the number of persons who have never worked that type problem since leaving high school. Under the heading "weight" is a value for each problem obtained by giving each check in the "often" column a value of 2, each check in the "seldom" column a value of 1, and each check in the "never" column a value of 0, the "weight" tabulated being the sum of these values.

Of the 137 type problems, "Checked a bill to see if all items were correct" ranks highest in weight. Table II shows that of the 101 people marking the check list, 33 indicate that they solved this problem often, 13, seldom, and none that he had never done it. The weight for this problem, then,

TABLE II

137 Type Problems Used Often, Seldom, or Never in Ordinary Occupations by 101 People in Kansas Communities

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
1	Checked a bill received to see if all items were correct	83	18	0	194
2	Made out or signed receipts	65	30	6	160
3	Kept record of money spent in home or business	58	35	8	151
4	Made out a statement or a bill	62	26	13	150
5	Solved problems using addition, subtraction, multiplication, or division of decimals	56	38	7	150
5	Checked bank balance sheet to see if balance was correct	55	40	6	150
7	Used per cent to figure interest problems	49	42	10	140
8	Multiplied mixed numbers	46	45	10	137
9	Figured discount given for cash	44	41	16	129
10	Added fractions	38	48	15	124
11	Found what per cent one number is of another	36	51	14	123
12	Read or written numbers in the millions or billions	35	49	17	119
13	Measured length of any object and expressed the answer decimally	28	56	17	112
14	Used a square	38	33	30	109
15	Reduced mixed numbers to improper fractions	28	46	27	102
16	Checked personal or property tax receipt to see if correct	26	45	29	101
17	Used compass to draw a circle	26	42	31	98

TABLE II--Continued

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
18	Figured cost of owning a car or home including the original cost, upkeep, interest on investment, taxes, and depreciation	23	51	27	97
19	Observed or interpreted graphs in papers, magazines, or books	27	42	32	96
20	Made out budget of expenditures over a given period of time	23	49	29	95
21½	Drawn a plan or map to scale .	19	54	36	92
21½	Figured interest rate when buying on the installment plan	24	44	33	92
23	Raised fractions to a higher denominator	23	44	34	90
24	Figured expense of keeping an old car and compared it with the expense involved in buying a new one	16	55	30	87
25	Figured a lumber bill in which you had to compute the number of board feet	25	35	41	85
26	Used the value of π in figuring the area of a circle .	21	41	39	83
27	Investigated life insurance to the extent of learning the different kinds of policies.	12	58	31	82
28	Figured the number of bushels of grain in a bin	23	35	33	81
29	Figured the number of gallons in a given number of cubic feet	17	46	38	80
30½	Used table of fractional equivalents as a short method in multiplying	24	30	47	78

TABLE II--Continued

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
30½	Figured the loan or cash value of an insurance policy	9	60	32	78
32	Solved problems representing the unknown with x	14	44	43	72
33	Read a light, gas, or water meter and figured the cost of service	19	33	49	71
34½	Measured wood by the rank or cord	16	34	51	66
34½	Solved a simple equation	11	44	46	66
36	Observed geometric designs in nature, conscious that they were geometric	16	32	53	64
37	Found area of a triangle using the rule that the area of a triangle equals $\frac{1}{2}$ the base x height	14	35	52	63
38	Measured the area of an irregular plot of ground in acres	14	33	44	61
39	Figured increase or decrease in taxes using mills as the rate	10	38	53	58
40½	Used per cent to figure bank discount	12	33	56	57
40½	Constructed parallel lines	7	43	51	57
42	Bisected a given line	6	42	53	54
43	Solved a problem of this type: One boy can do a piece of work in 5 days; another can do the same work in 7 days. How long will it take to do the work if they work together?	6	40	55	52
45	Used the theory that a triangle is rigid by making triangular braces	15	21	65	51
45	Used a protractor	9	33	60	51

TABLE II--Continued

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
45	Solved a mixture problem, as: How much water must be added to a quart of 20% solution to reduce it to a 5% solu- tion	8	35	58	51
47½	Used negative numbers	10	29	62	49
47½	Found the volume of a cube	10	29	62	49
49	Used the type of reasoning em- ployed in geometry	8	31	62	47
50½	Figured compound interest as computed by life insurance or investment companies	7	33	62	46
50½	Solved equation of the type: $4x - 5 - 2x = 23$	6	34	61	46
52	Drawn or constructed a graph	7	31	63	45
53	Used metric system of measure- ment	11	22	68	44
55	Used a drawing board and a T square	11	19	71	41
55	Figured the length of a rafter for a house with a definite pitch	7	27	67	41
56	Figured the cost of making a concrete wall	7	27	67	41
57	Used the terms "hypotenuse" and "legs" of a right trian- gle	8	23	70	39
59	Found the square root of a num- ber that has at least three digits	6	28	69	39
59	Figured the cost of excavating at a given cost per cubic yard	6	28	69	39
59	Used median, mode, or average.	4	30	67	38
62	Made two lines parallel to each other by making them perpendicular to the same line	7	21	73	35

TABLE II--Continued

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
62	Constructed an angle equal to a given angle	6	23	72	35
62	Used per cent to interpret graphs	8	19	74	35
65	Used the term "converse" . . .	6	21	74	33
65	Found the center of a circle by drawing two chords across the circle and erecting a perpendicular bisector of the chords	6	21	74	33
65	Solved fractional equations . .	8	17	76	33
67	Changed Fahrenheit to Centigrade or vice versa	5	22	74	32
68	Used the word "concurrent" . .	6	17	78	29
69½	Solved a problem in proportion which made use of the fact that the product of the means equals the product of the extremes	7	14	80	28
69½	Constructed a triangle similar to a given triangle	4	20	77	28
71½	Found a leg of a right triangle when given hypotenuse and the other leg	6	15	80	27
71½	Found the diagonal of a square or rectangle	7	13	81	27
73½	Drawn a regular hexagon	3	20	78	26
73½	Figured calorie content in comparing food prices or planned a meal having correct number of calories	7	12	82	26
75	Multiplied positive and negative numbers	3	19	80	25
76½	Used the term "exponent"	3	17	81	23
76½	Found the area of a trapezoid. .	5	13	83	23
79½	Solved a decimal equation . . .	5	12	84	22
79½	Found the surface of a sphere. .	5	12	84	22

TABLE II--Continued

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
79½	Cleared fractions and solved equation	3	16	82	22
79½	Measured the height of an object using its shadow in comparison with an object of known height and its shadow.	3	16	82	22
82	Found the number of degrees in the third angle of a triangle when two angles were given	3	15	83	21
83½	Added several algebraic numbers	4	12	85	20
83½	Divided a line segment into equal parts by proportion .	1	18	82	20
86	Solved problems concerning the 30°, 60°, 90° triangle using the fact that the short leg is equal to one-half the hypotenuse	4	11	86	19
86	Done landscaping which involved construction of geometric figures	4	11	86	19
86	Reduced algebraic fractions to lowest terms	3	13	85	19
89½	Constructed a complement or supplement of a given angle.	1	15	85	17
89½	Figured proportion by algebra.	2	13	86	17
89½	Figured horse power needed to pull an object of given weight up a given grade . .	3	11	87	17
89½	Proved two triangles were congruent	1	14	85	17
92	Removed a common factor from a polynomial	3	10	88	16
94½	Solved a literal equation . .	4	7	90	15
94½	Used tables to extract square or cube root of numbers . .	3	9	89	15

TABLE II--Continued

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
94½	Solved problem which made use of the fact that if two quantities are in proportion, they are in proportion by addition or subtraction . . .	3	9	90	15
94½	Found the least common multiple of several expressions . . .	2	11	88	15
98	Changed fractions to equivalent fractions	4	6	91	14
98	Referred to a variable approaching a limit	3	8	90	14
98	Changed algebraic fraction in form by changing the sign of the fraction	2	10	89	14
102	Solved a problem which involved the use of a tangent to a circle	1	11	89	13
102	Subtracted polynomials	3	7	91	13
102	Multiplied a polynomial by a monomial	3	7	91	13
102	Squared a binomial	3	7	91	13
102	Solved a problem using the principle that angles whose sides are parallel each to each are either equal or supplementary	2	9	90	13
109½	Constructed a tangent from a point to a circle	1	10	90	12
109½	Worked problems in which it was necessary to use the principle that the exterior angle of a triangle equals the sum of the two non-adjacent interior angles	2	8	91	12
109½	Multiplied monomials	3	6	92	12
109½	Multiplied algebraic fractions	4	4	93	12

TABLE II--Continued

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
109½	Circumscribed a circle about a regular polygon	1	10	90	12
109½	Used the fact that the medians, altitudes, bisectors of the angles, or perpendicular bisectors of the sides of a triangle meet in a point	1	10	90	12
109½	Found the distance to an inaccessible point by the use of geometry	1	10	90	12
109½	Solved a set of linear equations for x and y	3	6	92	12
109½	Divided by long division in algebra	3	6	92	12
109½	Simplified a complex fraction.	2	8	91	12
115½	Found the area of a trapezoid, substituting the median for one-half the sum of the bases	2	7	92	11
115½	Proved two lines were parallel by proving the alternate interior angles to be equal	2	7	92	11
119½	Found the cube root of a monomial expression	2	6	93	10
119½	Multiplied monomials together which had negative exponents	2	6	93	10
119½	Added numerators and denominators in a series of equal ratios and set them equal to any one of the ratios	1	8	92	10
119½	Factored a perfect square	3	4	94	10
119½	Found the area of a triangle by using the formula $A = s(s-a)(s-b)(s-c)$	1	8	92	10
124½	Divided a polynomial by a monomial	2	5	94	9
124½	Used formula for solving quadratic equations	3	3	95	9

TABLE II--Concluded

Rank	Problem	Solved			Weight
		Often	Seldom	Never	
124½	Proved two line segments were equal by proving they were corresponding parts of congruent triangles	1	7	93	9
124½	Factored the sum of two cubes.	1	7	93	9
128	Used table of sines, cosines, and tangents	3	2	96	8
128	Solved quadratic equation . .	3	2	96	8
128	Found the number of degrees in a polygon of a given number of sides	1	6	94	8
131½	Added and subtracted radicals.	2	3	96	7
131½	Constructed a graph from a linear equation	3	1	97	7
131½	Measured an angle which was formed by a tangent and a chord by measuring one-half the intercepted arc	1	5	95	7
131½	Used the term "apothem" . . .	1	5	95	7
134	Solved a mean proportion problem in which the tangent to a circle was the mean proportion between the secant and its external segment	1	3	97	5
136	Solved problems in which the product of the segments of one of the intersecting chords was equal to the product of the segments of the other	1	2	98	4
136	Had occasion to raise a number to the zero power	1	2	98	4
136	Made a graph of a quadratic equation	1	2	98	4

is 184, which was determined by doubling the 83 in the "often" column and adding the 18 in the "seldom" list. "Made out or signed receipts" ranks second in weight. Of the 101 people, 65 solved this problem often; 30, seldom; and 6, never; the weight for this item totals 160.

Problems Solved Often

Of the problems solved often, "Checked a bill to see if all items were correct" ranks first with a frequency of 83; "Made out or signed receipts" with a frequency of 65 ranks second; "Made out a statement of bill" which has a frequency of 62 ranks third; and "Kept record of money spent in home or business" with a frequency of 59 is fourth. These frequencies indicate how many of the 101 persons marking the check list used these mathematical operations many times. The frequencies are also approximately equivalent to the per cent of people solving the problems often, as there were 101 people who marked the check list.

The first four type problems in Table II, which have the greatest frequencies in the "often" column, are problems dealing with home expenditures. Considerable time and attention, therefore, should be given in school to this type of problems. In general, problems involving simple arithmetical operations are the type most often solved by the people checking the list. The frequencies in the "often" solved

column indicate that much of the mathematics higher than arithmetic is used often by only a small per cent of the people.

Study of the individual check lists showed that many of the algebra and geometry problems were used often by only four persons: the mathematics teacher, a barber who had studied engineering, the physician, and the contractor. There was none of the problems on the check list that was not used often by at least one individual, since one of the two teachers checking the list is a teacher of mathematics, who checked almost all of the problems with a "2".

Problems Seldom Solved

The frequencies of the problems seldom solved by the 101 people of Kansas who were interviewed are listed in the second column of Table II. Frequencies of this classification range from 60 to 2. Of the 157 problems listed, every problem was solved occasionally by at least two persons.

"Figured the loan or cash value of an insurance policy" ranks highest in the "seldom" column with a frequency of 60. It is probably true that most people need to investigate the loan or cash value of an insurance policy only a few times. Nevertheless, an understanding of an insurance policy is important, although a great majority of the people examine their policies only occasionally.

Fifty-eight per cent of the people "Investigated life insurance to the extent of learning the different kinds of policies"; this item ranked second highest in the "seldom" column. Thus again it may be that life insurance, though investigated seldom, is of much importance to the majority. "Measured the length of any object and expressed the answer decimally" ranks third in the "seldom" column with a frequency of 58, and "Figured the expense of keeping the old car and compared it with the expense involved in buying a new one" ranks fourth in this column, being considered by 50 per cent of those marking the check list.

Problems Never Solved

From the third column of figures in Table II, under the heading "never", one may determine how many of the type problems given in the check list have never been solved by the people marking it. As a rule the problems with the higher frequencies indicate that more people have never solved those problems. This usually means that these problems are of less importance, although there may be exceptions. For example, the use of tables in the extraction of square root, although not used by 89 per cent of the group interviewed, represents a method which is rather important, especially so since little emphasis is now being placed upon the actual mathematical processes involved in the extraction of roots.

The problems that have never been solved by the greatest number of people are: "Made a graph of a quadratic equation," "Had occasion to raise a number to the zero power," and "Solved problems in which the segments of one of the intersecting chords was equal to the product of the other." These three problems have never been solved by 93 per cent of the 101 people interviewed. The "never" column shows that the more difficult problems are the least used, and that much of the algebra and geometry above the simplest processes is never used by a great majority of those marking the check lists. "Checked a bill received to see if all items are correct" is the only problem of the 137 given in the check list to be solved by all those interviewed.

Weight

From the fourth column in Table II, under the heading "weight", one may read a relative value for the usability of the problems. It is the writer's opinion that the problems solved often should receive more weight than those solved occasionally. For this reason the frequencies in the "often" column were assigned a weight of "2", while those in the "seldom" column were given a weight of "1", and those in the "never" column were given a weight of "0". The possible maximum total weight of any problem would thus be 202. The last column of figures, Table II, shows the weight determined on

this basis for each problem in the check list.

The total weights of the problems range from 184 to 4. The six highest ranking problems according to weight are: "Checked a bill received to see if all items were correct," weight, 184; "Made out or signed receipts," weight, 160; "Kept a record of money spent in home or business," weight, 151; "Made out a statement or bill," weight, 150; "Solved problems using addition, subtraction, multiplication, or division of decimals," weight, 150; and "Checked a bank balance sheet," weight, 150.

The three problems ranking lowest with respect to weight are: "Solved problems in which the product of the segments of one of the intersecting chords was equal to the product of the segment of the other," "Had occasion to raise a number to the zero power," and "Made a graph from a quadratic equation." Each of these problems obtained a weight of 4 from the checks marked by the persons interviewed. The low rank in the weight column of some of the items indicates that perhaps more effective work could be done in teaching people the values and possible uses of these problems little used outside of the classroom. However, it is quite possible that many people never need to solve these problems receiving a small "weight" value and ranking high in the "never" column. An analysis of the activities of the 101 people checking the list would be necessary to determine

whether failure to use certain processes resulted from the inability to apply the principles involved to the situations encountered, or from lack of opportunities in which certain mathematical techniques could be applied.

Summary

The problems solved often, seldom, and never, by 101 persons representing thirty-eight different occupations are tabulated in Table II, pages 11 to 19, together with the weights assigned to their replies by the writer.

This table indicates that arithmetic problems are used much more frequently than the problems involving algebra and geometry, for even the simpler types of algebra and geometry problems are used frequently by less than one-half of the people interviewed, and problems involving the more complex processes of algebra and geometry are used by no more than 10 per cent of the persons marking the check list.

CHAPTER III

INTERPRETATION AND CLASSIFICATION OF DATA

The purpose of this chapter is to determine which of the problems in each of the twenty groups or sections of the check list are most frequently solved. In addition, there will be an attempt to determine the most used areas of general mathematics, algebra, and geometry. This analysis should be valuable as a basis for selecting the subject matter to be included in high-school mathematics textbooks, especially the textbooks in general mathematics. If pertinent content can be incorporated into general mathematics courses, thus offering to high-school boys and girls the most usable material, they will be better prepared to solve the problems which are likely to arise in their lives.

The 137 type problems checked by 101 people were ranked according to the total weight of each problem in Table II of the preceding chapter. In the current chapter all problems are classified, ranked, and discussed with respect to the division of mathematics in which they are studied, i. e., arithmetic, algebra, and geometry. The problems falling within each of these major mathematical divisions are grouped under the following subtitles:

ARITHMETIC: Measurement, Fractions and Decimals, Percentage, Home Expenditures, Insurance, Areas, and Miscellaneous.

ALGEBRA: Graphs, Factoring, Fractions, Equations, Simple Processes, and Miscellaneous.

GEOMETRY: Construction, Triangles, Circles, Polygons, Parallels, Proportion, and Miscellaneous.

This chapter consists of a tabulation and analysis of the problems contained in each of the twenty sections of the check list. Each section of the chapter follows the same pattern; viz., a table presents the problems in order of their assigned "weight"; the problems having sufficient weight to warrant inclusion in a general mathematics course are pointed out; and those problems which are little used by the 101 people interviewed are mentioned. Such a detailed analysis may become somewhat monotonous to the reader unless he is interested in the reaction to each specific problem included in the check list.

Arithmetic

Measurement. Of the problems involving measurement, "Measured length of any object and expressed the answer decimally" ranks first, having a weight of 112. "Drawn a plan or map to scale" ranks second with a weight of 92. Table III, page 26, shows that the third, fourth, and fifth ranking measurement problems are of almost equal weight, weighing 85, 81, and 80, respectively.

From this table it can readily be seen that problems involving measurement are important to many people. The

TABLE III

Weight of Problems Involving Measurement Solved Often, Seldom, and Never--Determined from the Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Measured length of any object and expressed the answer decimally	112
2	Drawn a plan or map to scale	92
3	Figured a lumber bill in which you had to compute the number of board feet . . .	85
4	Figured the number of bushels of grain in a bin	81
5	Figured the number of gallons in a given number of cubic feet	80
6	Measured wood by the rank or cord	66
7	Measured the area of an irregular plot of ground in acres	61
8	Used metric system of measurement	44
9	Figured the length of a rafter for a house with a definite pitch	41

last two problems in this table, "Used the metric system of measurement" and "Figured the length of a rafter . . .", seemed to occur rather infrequently.

Fractions and Decimals. The six type problems on the check list pertaining to the use of fractions and decimals appear to be used quite often. The average weight of the problems in this group ranks second high among the average weights of the twenty groups. In Table IV, page 29, the use of decimals outweighs the use of common fractions, for "Solved problems using addition, subtraction, multiplication,

TABLE IV

Weight of Problems Involving Fractions and Decimals, Solved Often, Seldom, and Never--Determined from the Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Solved problems using addition, subtraction, multiplication, or division of decimals .	150
2	Multiplied mixed numbers, as $4\frac{1}{2}$ times $5\frac{1}{2}$.	137
3	Added fractions, as $\frac{3}{16} + \frac{7}{8} + \frac{2}{3}$. . .	124
4	Reduced mixed numbers to improper fractions.	102
5	Raised fractions to a higher denominator .	90
6	Used table of fractional equivalents as a short method of multiplying	78

and division of fractions" ranks highest with a weight of 150. In this group, "Used the table of fractional equivalents as a short method of multiplying" ranks least with a weight of 78. The high frequency reported for the solution of problems concerning common fractions and decimals suggests that intensive work should be done in the instruction of this section of arithmetic in the classroom.

Percentage. Among the percentage problems, "Used per cent to figure interest problems" ranked first; it has a weight of 138. In Table V, page 30, "Figured discount given for cash" ranks second, and "Found what per cent one number is of another" is third. These problems seem of almost equal importance; the former having a weight of 129; the

TABLE V

Weight of Percentage Problems Solved Often, Seldom, and
Never--Determined from Replies of 101 People

Rank	Problem	Weight
1	Used per cent to figure interest problems .	138
2	Figured discount given for cash	129
3	Found what per cent one number is of another	123
4	Figured the interest rate when buying on the installment plan	92
5	Used per cent to figure bank discount . . .	57
6	Used per cent to interpret graphs	35

latter, a weight of 123. "Figured the interest rate when buying on the installment plan" is rather important, judging from the frequency of its occurrence listed in Table II, which shows that 68 per cent of the people interviewed had had occasion to use the process. Therefore, the inclusion in the general mathematics textbook of a section on installment buying seems advisable.

The use of percentage in the interpretation of graphs ranks last among the problems given in Table V. This problem has a weight of only 35. The status of this problem is explicable when one considers that most people make a visual comparison of the ratios presented by means of a graph rather than thinking in terms of percentage.

Home Expenditures. Ranking highest in weight of the 137

TABLE VI

Weight of Home Expenditures Problems Solved Often, Seldom
and Never--Determined from Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Checked a bill received to see if all items were correct	184
2	Made out or signed receipts	160
3	Kept record of money spent in home or busi- ness	151
4	Made out a statement or a bill	150
5	Made out a budget of expenditures over a given period of time	95
6	Read a light, gas, or water meter and fig- ured the cost of service	71
7	Figured calorie content in comparing food prices, or planned a meal having correct number of calories	26

problems included in the check list are the first four prob-
lems listed in Table VI, above. "Checked a bill to see if
all items were correct" ranks first with a weight of 184;
"Made out or signed receipts" ranks second with a weight of
160; "Kept a record of money spent in home or business" ranks
third with a weight of 151; and "Made out a statement or
bill" ranks fourth with a weight of 150. Many people also
make out budgets of expenditures and read light, gas or
water meters. Figuring calorie content of food seems to be
of relatively little importance to most people, as this prob-
lem received a weight of only 26. It is significant that the

problems of this group have the highest average of all the groups of problems on the check list. It is evident that this type of problem should receive much emphasis in the general mathematics course.

Insurance. Table VII, below, shows that among the insurance problems "Investigated life insurance to the

TABLE VII

Weight of Insurance Problems Solved Often, Seldom, and Never--Determined from Replies of 101 People

Rank	Problem	Weight
1	Investigated life insurance to the extent of learning the different kinds of policies .	82
2	Figured the loan or cash value of an insurance policy	78
3	Figured compound interest as computed by life insurance or investment companies .	48

extent of learning the different kinds of insurance policies" ranks highest with a weight of 82. "Figured the loan or cash value of an insurance policy" is second with a weighting of 78. The fact that 70 per cent of the people needed at some time to solve each of these two type problems is sufficient evidence to warrant a careful study of insurance in high-school mathematics. The necessity of figuring compound interest as computed by life-insurance or investment companies received the least weight of the problems in

this group.

Areas. The most used problems in computing areas were "Used the value of π in figuring the area of a circle," weighing 83; and "Found the area of a triangle using the rule that the area of a triangle equals one-half the base times the height," weighing 63. The areas of spheres and trapezoids are not frequently computed by the people interviewed, as shown by Table VIII, below. The weight received

TABLE VIII

Weight of Problems in Computing Areas, Solved Often, Seldom, and Never--Determined from Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Used the value of π in figuring the area of a circle	83
2	Found the area of a triangle using the rule that the area of a triangle equals $\frac{1}{2}$ the base times the height	63
3	Found the area of a trapezoid	23
4	Found the surface of a sphere using the formula $S = 4\pi r^2$	22

by the first problem listed in this table indicates that emphasis should be given the study of circles.

Miscellaneous Arithmetic. Eleven arithmetic problems are included in Table IX, which ranks the problems included in the miscellaneous arithmetic section of the check list.

TABLE IX

Weight of Miscellaneous Arithmetic Problems Solved Often, Seldom, and Never--Determined from Replies of 101 People

Rank	Problem	Weight
1	Checked a bank balance sheet to see if balance was correct	150
2	Read or written numbers in the millions or billions	119
3	Checked a personal or property tax receipt to see if correct	101
4	Figured the cost of owning a car or home including the original cost, upkeep, interest on investment, taxes, and depreciation	97
5	Figured expense on keeping an old car and compared with the expense involved in buying a new one	87
6	Figured increase or decrease in taxes using mills as the rate	58
7	Found the volume of a cube	49
8	Figured the cost of making a concrete wall	41
9 $\frac{1}{2}$	Found the square root of a number that has at least three digits	38
9 $\frac{1}{2}$	Figured the cost of excavating at a given cost per cubic yard	39
11	Figured horse power needed to pull an object of given weight up a given grade	17

"Checked a bank balance sheet" and "Read or written numbers in the millions or billions" rank at the top of this group. These items weigh 150 and 119, respectively. The weightings given for the next three problems in this tabulation are sufficient that problems of this type should merit considerable attention in secondary-school mathematics courses.

Algebra

Graphs. Problems concerning graphs make up an important section of algebra. It is significant to note that 69 per cent of the people interviewed "Observed or interpreted graphs in papers, magazines, or books." This item not only ranked first among the problems listed in Table X, below, but also received the greatest weight of any of the problems involving algebraic processes. It is interesting to note that few of the 101 people constructed any type of graphs, and that virtually none constructed graphs from either linear or quadratic equations. However, the writer

TABLE X

Weight of Problems Concerning Graphs, Solved Often, Seldom, and Never--Determined from Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Observed or interpreted graphs in papers, magazines, or books	96
2	Drawn or constructed a graph	45
3	Used median, mode or average	38
4	Constructed a graph from a linear equation	7
5	Made a graph of a quadratic equation	4

feels that it is probably necessary to teach construction of graphs in order that graphs may be correctly interpreted.

Factoring. The average weight of the problems listed in

Table XI ranks lower than that of any other group of problems considered in this study. Factoring, it appears, is little used except in scientific work and in teaching mathematics. For that reason, it seems unnecessary to devote so much time to factoring as it now receives in the classroom.

TABLE XI

Weight of Factoring Problems Solved Often, Seldom, and Never--Determined from Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Removed a common factor from a polynomial .	16
2	Factored a perfect square : : : : :	10
3	Factored sum of two cubes : : : : :	9

Algebraic Fractions. Algebraic fractions seem to be unused by about 90 per cent of the people. It is probable that little time should be given to this type of work in the field of general mathematics. Weights of the five problems ranked in Table XII, page 37, range from 12 to 19.

Equations. About 60 per cent of the people marking the check list had "Solved problems representing the unknown with x" and "Solved a simple equation." Equations of forms higher than the simple equation are little used. Table XIII, page 37, shows that quadratic equations are seldom used, for

TABLE XII

Weight of Problems Involving Algebraic Fractions, Solved Often, Seldom, and Never--Determined from Replies of 101 People

Rank	Problem	Weight
1	Reduced algebraic fractions to lowest terms.	19
2 $\frac{1}{2}$	Changed fractions to equivalent fractions .	14
2 $\frac{1}{2}$	Changed an algebraic fraction in form by changing the sign of the fraction . . .	14
4 $\frac{1}{2}$	Multiplied algebraic fractions	12
4 $\frac{1}{2}$	Simplified a complex algebraic fraction . .	12

TABLE XIII

Weight of Problems Involving Equations Solved Often, Seldom, and Never--Determined from Replies of 101 People

Rank	Problem	Weight
1	Solved problems representing the unknown with x	72
2	Solved a simple equation, as $2x = 12$. . .	66
3	Solved equations of the type: $4x - 5 = 2x + 23$	46
4	Solved fractional equations, as $x/7 = 1$. .	33
5	Changed Fahrenheit to Centigrade or vice versa	32
6 $\frac{1}{2}$	Cleared fractions and solved equation . .	23
6 $\frac{1}{2}$	Solved decimal equation	22
8	Solved a literal equation	15
9	Solved a set of linear equations for x and y	12
10	Used formula for solving quadratic equation.	9
11	Solved quadratic equation	8

the weights of the processes involving quadratic equations are 9 and 8. In fact, 96 per cent of the people had never solved a quadratic equation since leaving high school. These facts indicate that relatively more emphasis should be given problems involving simple equations, for they have a more practical value. Furthermore, the replies indicate that quadratic equations have not been presented to the student in a manner for him to perceive their usability.

Simple Algebraic Processes. In Table XIV, below, which ranks the problems dealing with simple algebraic processes, "Multiplied positive and negative numbers" is first with a weight of 25. "Added several algebraic numbers" ranks

TABLE XIV

Weight of Problems Dealing with Simple Algebraic Processes,
Solved Often, Seldom, and Never--Determined from
the Replies of 101 People

Rank	Problem	Weight
1	Multiplied positive and negative numbers .	25
2	Added several algebraic numbers	20
4	Subtracted polynomials	13
4	Multiplied a polynomial by a monomial . .	13
4	Squared a binomial	13
6½	Multiplied monomials	12
6½	Divided by long division in algebra . . .	12
8	Multiplied monomials together which had negative exponents	10
9	Divided a polynomial by a monomial . . .	9

second with a weight of 20. Other simple processes seem to be little used by the 101 people who marked the check list. It is apparent from the weights of these problems that algebra beyond the very elementary processes is never used by more than 10 per cent of the people interviewed.

Miscellaneous Algebra. According to Table XV, which lists the miscellaneous algebra problems, the weights of the

TABLE XV

Weight of Miscellaneous Algebra Problems Solved Often, Seldom, and Never--Determined from Replies of 101 People

Rank	Problem	Weight
1	Solved a problem of this type: One boy can do a piece of work in five days; another can do the same work in seven days. How long will it take to do the work if they work together?	52
2	Solved a mixture problem, as: How much water must be added to a quart of 20% solution to reduce it to a 5% solution?	51
3	Used negative numbers	49
4	Used the term "exponent"	23
5	Figured proportion by algebra	17
6 $\frac{1}{2}$	Found the least common multiple of several expressions	15
6 $\frac{1}{2}$	Used table to extract square or cube root of numbers	15
8	Found the cube root of a monomial expression	10
9	Used table of sines, cosines, and tangents .	8
10	Added and subtracted radicals	7
11	Had occasion to raise a number to the zero power	4

first three problems are significant. About 45 per cent of the people have solved these three types of problems: (1) "One boy can do a piece of work in five days; another can do the same work in seven days. How long will it take them both to do the work together?" (2) mixture problems, and (3) "Used negative numbers." "Had occasion to raise a number to the zero power" ranks last in this group of problems, and is one of the three lowest ranking problems of all those on the check list. It has a weighting of only 4 and was used by only 2 per cent of those interviewed--the mathematics teacher was the only one who had worked this type problem frequently. Although their weights are somewhat higher, the remaining problems of this group have little importance in the high-school mathematics curriculum.

Geometry

Construction. The geometry problems involving construction are ranked in Table XVI, page 41. "Construction of parallel lines" heads the list with a weight of 57. This operation was used by 50 per cent of the people interviewed. "Bisected a given line accurately" ranks second with a weight of 54. The simple problems of construction in geometry seem to be applicable, but problems of a more complex type, such as "Constructed a tangent from a point to a circle." seem to be of very little practical value for they have weights less

TABLE XVI

Weight of Construction Problems Solved Often, Seldom, and Never--Determined from the Replies of 101 People

Rank	Problem	Weight
1	Constructed parallel lines	57
2	Bisected a given line accurately	54
3	Constructed an angle equal to a given angle.	35
4	Constructed a triangle similar to a given triangle	28
5	Drawn a regular hexagon	26
6	Constructed a complement or supplement of a given angle	17
7 $\frac{1}{2}$	Constructed a tangent from a point to a cir- cle	12
7 $\frac{1}{2}$	Circumscribed a circle about a regular poly- gon	12

than 20.

Triangles. According to Table XVII, page 42, a tabulation of the problems concerning triangles, "Used the theory that a triangle is rigid by the use of triangular braces" ranks first with a weight of 51, and is used by 35 per cent of the people interviewed. "Used the term 'hypotenuse' and 'legs' of a right triangle" ranks second with a weight of 39, and was used by only 30 per cent of the people replying. It is probably true that if a practical presentation of this type problems had been given to the students, a greater number of persons might have indicated their application, for they involve principles that have a wide application. The

TABLE XVII

Weight of Problems Concerning Triangles, Solved Often, Seldom
and Never--Determined from the Replies of 101 People

Rank	Problem	Weight
1	Used the theory that a triangle is rigid by making triangular braces	51
2	Used the terms "hypotenuse" and "legs" of a right triangle	39
3	Found a leg of a right triangle when given hypotenuse and the other leg	27
4	Found the number of degrees in the third angle of a triangle when two angles were given	21
5	Solved problems concerning the 30° , 60° , 90° triangle using the fact that the short leg is equal to one-half the hypotenuse	19
6	Proved two triangles were congruent	16
7 $\frac{1}{2}$	Worked problems in which it was necessary to use the principle that the exterior angle of a triangle equals the sum of the two non-adjacent interior angles	12
7 $\frac{1}{2}$	Used the fact that the medians, altitudes, bisectors of the angles, or the perpendicular bisectors of the sides of a triangle meet in a point	12
9	Proved two line segments were equal by proving they were corresponding parts of congruent triangles	9

item, "Proved that line segments were equal by proving they were corresponding parts of congruent triangles," ranked last with a weight of only 9, which shows that only 5 per cent of the people represented in the occupations studied are ever called upon to work problems of congruency.

Circles. Of the five problems concerning circles, listed in Table XVIII, "Used a compass to draw a circle" has a weight of 98, and has been performed by 70 per cent of the people marking the check list. However, it appears that very little use is made of the problems involving chords, tangents, and measurement of angles, for the last four problems in this section have very small weight values.

TABLE XVIII

Weight of Problems Concerning Circles, Solved Often, Seldom, and Never--Determined from the Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Used compass to draw a circle	98
2	Found the center of a circle by drawing two chords across the circle and erecting a perpendicular bisector of the chords . .	33
3	Solved a problem which involved the use of a tangent to a circle	13
4	Measured an angle which was formed by a tangent and a chord by measuring one half the intercepted arc	7
5	Solved problems in which the product of the segments of one of the intersecting chords was equal to the product of the segments of the other	4

Polygons. There is very little weight accorded to problems relative to polygons. "Found a diagonal of a square or rectangle" ranks first with a weight of 27, having been used by only 20 per cent of the people marking the check list.

TABLE XIX

Weight of Problems Concerning Polygons Solved Often, Seldom,
and Never--Determined from the Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Found the diagonal of a square or a rectan- gle	27
2	Found the area of a trapezoid, substituting the median for one-half the sum of the bases	11
3	Found the number of degrees in a polygon of a given number of sides	6
4	Used the term "apothem"	7

The other three type problems in this group have even less practical value, as they were used by less than 7 per cent of those interviewed.

Parallels. From Table XX, which lists processes con-

TABLE XX

Weight of Problems Concerning Parallels Solved Often, Seldom,
and Never--Determined from the Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Made two lines parallel to each other by mak- ing them perpendicular to the same line .	35
2	Solved a problem using the principle that an- gles whose sides are parallel each to each are either equal or supplementary . . .	13
3	Proved two lines were parallel by proving the alternate interior angles to be equal.	11

TABLE XXI

Weight of Proportion Problems Solved Often, Seldom, and
Never--Determined from the Replies of 101 People

<u>Rank</u>	<u>Problem</u>	<u>Weight</u>
1	Solved a problem in Proportion in which you used the fact that the product of the means equals the product of the extremes .	38
2	Measured the height of an object using its shadow in comparison with an object of known height and its shadow	22
3	Divided a line segment into equal parts by proportion	20
4	Solved a problem which made use of the fact that if two quantities are in proportion, they are in proportion by addition or subtraction	15
5	Found the distance to an inaccessible point by the use of geometry	12
6½	Added numerators and denominators in a series of equal ratios and set them equal to any one of the ratios	10
6½	Divided a line externally, internally, or harmonically	10
8	Solved a mean proportion problem in which the tangent to a circle was the mean proportion between the secant and its external segment	5

cerning parallels, "The construction of parallel lines," is the only problem having sufficient weight to merit consideration. It has a weight of 35 and was used by about 30 per cent of the people, thus pointing out again that the simple construction problems are of importance to only one-third of

the people interviewed.

Proportion. The weight of problems concerning proportions is very low; the highest, "Solved a problem in proportion using the fact that the product of the means equals the product of the extremes," weighing only 28, was used by only 20 per cent of the people marking the check list.

Table XXI, page 45, shows that the second, third, fourth, and fifth problems, which are often considered practical, are, in reality, seldom used by the students of geometry after they have graduated from high school.

Miscellaneous Geometry. Of the ten miscellaneous geometry problems listed in Table XXII, "Used a square" ranks first with a weight of 109. It was used by 70 per cent of the people interviewed. About one-half of these people state that they have "Observed geometric designs in nature, conscious that they were geometric." "Used the protractor" has enough weighting to warrant instruction in its use to all students. "Used the type of reasoning employed in geometry" has a weight of 47, and was used by 40 per cent of the group replying. The checking of this item indicates that this may be one of the important values to be gained from a course in geometry, although it seems to be of less importance than is often supposed by many mathematicians.

Evidently, the use of the drawing board has some practical value, but, apparently, only one-fourth of the

TABLE XXII

Weight of Miscellaneous Geometry Problems Solved Often, Seldom, and Never--Determined from Replies of 101 People

Rank	Problem	Weight
1	Used a square	100
2	Observed geometric designs in nature, conscious that they were geometric	64
3	Used a protractor	51
4	Used the type of reasoning employed in geometry; namely, using material given, building up a proof, then drawing conclusion	47
5	Used a drawing board and a T square	41
6	Used the term "converse"	33
7	Used the word "concurrent"	29
8	Done landscaping which involved construction of geometric figures	19
9	Referred to a variable approaching a limit	14
10	Found the area of a triangle using the formula $A = \frac{1}{2} s(s-a)(s-b)(s-c)$ where s equals $\frac{1}{2}$ the perimeter, and a , b , and c , are the sides	10

people reporting have use for the geometric terms, such as "converse" and "concurrent." Very few people are conscious of the value of geometric figures in doing landscaping, an item which has a weight of 19 and was used by only 15 per cent of the group. The last two problems in this table seem to be too technical for general use, for they were employed by only 10 per cent of the persons checking the list.

Comparison of the Weights of the Groups of Problems

A summary of the total and average weights given the replies of 101 persons to all the problems in each division of the check list is given in Table XXIII. The groups of problems are ranked in the order of the average weight of

TABLE XXIII

Total and Average Weight of Each Group of Problems--Determined from the Replies of 101 People in Kansas

<u>Group Classification</u>	<u>Total Weight</u>	<u>Problems in Group</u>	<u>Average Weight</u>
Home Expenditures	837	7	119.6
Common Fractions and Decimals	681	6	113.5
Percentage	574	6	95.7
Measurement	662	9	73.6
Insurance	206	3	68.7
Miscellaneous Arithmetic	795	12	66.2
Areas	191	4	47.7
Miscellaneous Geometry	417	10	41.7
Graphs	190	5	38
Circles	155	5	31
Algebraic Equations	337	11	30.6
Geometric Construction	241	8	30.1
Triangles	206	9	22.9
Miscellaneous Algebra	251	11	22.9
Parallels	59	3	19.7
Proportions	122	8	15.2
Algebraic Fractions	71	5	14.2
Simple Algebra Processes	127	9	14.1
Polygons	53	4	13.2
Factoring	35	3	11.7

all the problems in each division. From this table it may be seen that all problems classified under the heading "Home Expenditures" have a total weight of 837. The average weight for the seven problems of this group is 119.6. Problems requiring the use of common and decimal fractions rank second with an average weight of 113.5.

Problems in the following groups: Home Expenditures, Common Fractions and Decimals, Percentage, Measurement, and Insurance rank highest with respect to weightings which are determined by the frequency of their use among the 101 people interviewed. These weights are sufficiently high to indicate the importance of these type problems to the majority of people. The fact that mathematical processes involved in computing home expenditures rank highest shows that problems concerning the home are solved by many people.

Summary

The 137 type problems as given in Table II were considered in this chapter in their order of classification and grouping in the check list. The problems comprising each division were ranked in order of greatest weight. Each group of problems was then classified according to its place in the field of mathematics, i. e., arithmetic, algebra, or geometry. A total and average weight for each division shows the relative importance of the groups of

problems. However, the average frequency must not be considered as the only index in determining the importance of a single group or classification, for many problems rank very high and others very low within a given classification.

CHAPTER IV

FINDINGS AND RECOMMENDATIONS

Findings

This study proposes to determine the type problems in algebra, geometry, and arithmetic that are most used by high-school graduates in life situations in order to suggest material best suited from the standpoint of usability for a general mathematics course. Data were secured by means of a check list which was filled out by 101 people living in Kansas. The writer's interpretation of the value of their responses in selecting content for the general mathematics course is as follows:

1. Problems most frequently solved relate to the following topics:

- Home expenditures.
- Common and decimal fractions.
- Percentage.
- Measurement.
- Insurance.
- Installment buying.
- Banking.
- Taxation.
- Personal property.
- Areas of circles and triangles.
- Use of the square and protractor.
- Geometrical terms.
- Graphs.
- Simple algebraic equations.
- Simple geometric constructions.

These problems, therefore, should be thoroughly taught to

all pupils in a general mathematics course in high school.

2. Problems concerning insurance, drawing plans or maps to scale, problems using terms such as median, mode or average were seldom solved by many people. These problems, although solved occasionally by an individual, are not necessarily unimportant to him. Problems concerning insurance and drawing plans or maps to scale are of such a nature that solutions, though sought infrequently, are quite consequential. Problems involving statistical techniques, used extensively only in recent years, are probably more important than the data collected signify. The writer thinks those problems which are used only occasionally by many people should be given careful consideration in the secondary-school general mathematics course.

3. The problems which few people have solved since leaving high school involve factoring, complex problems dealing with polygons and algebraic fractions--in fact, almost all of the formal algebra above the simplest processes, and virtually all geometry with the exception of a few simple construction problems, a few of the simplest problems concerning the use of the square, the protractor, and the drawing board, and the use of a few simple geometrical terms. This finding may mean that this type of problems was not well-taught from the standpoint of practical application, or opportunities for the solution of these problems are

infrequently encountered by those people marking the check list. Since the people responding to the questionnaire were representatives from many different schools and had taken their work with many different mathematical instructors, it is the writer's opinion that the opportunities for using much of our formal mathematics seldom present themselves to persons in many occupations. It would seem that few of the complex problems of algebra and geometry are of sufficient importance to be valuable in a general mathematics course.

4. The interpretation of graphs is of importance to many people. Very few persons actually need to construct graphs. However, knowledge of the construction of graphs is valuable, though little used, in order that correct interpretation may be given to graphs.

5. The problems which are usually considered as belonging in the field of arithmetic have utility for most people. The use of common and decimal fractions is quite prevalent, and problems of business transactions are also important to almost everyone.

6. The findings of the writer agree in part with those of Bobbitt. Like Bobbitt, the writer found that such arithmetic and a moderate amount of algebra, particularly information regarding graphical interpretations, should be included in the practical mathematics course. However, he did not find quite as extensive a need for the vocabulary of geometry

as Bobbitt reported.

According to Kilzer and Thompson, the general mathematics textbooks give more attention to arithmetic and geometry than to algebra, but the writer found arithmetic to be of greater value than either algebra or geometry. The writer's material agrees with that of Thorndike, who found very little use for many things traditionally taught in algebra and emphasized the need for instruction in other algebraic processes, notably graphs and their interpretation.

Recommendations

The findings presented in this study of problems actually encountered in daily life warrant the following recommendations:

1. General mathematics should contain many problems concerning home expenditures, percentage, insurance, installment buying, measurement, areas, reading and constructing graphs, simple geometrical construction problems, and the very simplest algebraic equations.

2. Most students who intend to go into the less technical occupations of life can profit more from the materials presented in the practical general mathematics course than from traditional courses in algebra and geometry.

3. Those students who wish to continue in the scientific and related fields should be required to take algebra and

geometry, as general mathematics is not a substitute for algebra and geometry and their technical application. The content of textbooks in general mathematics should be analyzed and reorganized from the standpoint of usable material. There should be no attempt, as has been true in the past, to include an approximately equal sampling from the fields of arithmetic, algebra, and geometry.

4. General mathematics should be simplified and contain less algebra and geometry than is included in the present Kansas state-adopted textbook.

5. Teachers of general mathematics should carefully analyze the problems to see which are really practical, for the check-list returns indicate that many problems which mathematics instructors consider practical are little used. Not only should the problems be analyzed with care, but they should also be presented in such a manner that the student will have a thorough understanding of their practical value and be able to apply them to appropriate situations.

6. The textbook, Mathematics in Life, by Schorling and Clark, designed by its authors for the lower one-third of the class, contains most of the type problems receiving greatest weight in this investigation. Mathematics in Life is divided into eight units:

Measurement, Construction, Drawing to Scale, Percentage, Uses of Graphs, Wise Use of Money, Home and Business Arithmetic, and Formulas and Equations.

To this content, the writer would recommend the addition of a study of insurance, more study on fractions and decimals, and a section on installment buying. He would also advise that less time be devoted to formulas and equations than is provided for in this text.

7. Modern School Mathematics, by Schorling, Clark, and Smith, which is the textbook recently adopted by the State of Kansas for all students of general mathematics, is divided into the following eighteen sections:

Arithmetic in Everyday Life, Introduction to Algebra, Formulas, Dependence, Equations and Their Solution, Problem Solving, Measurement and Approximation, Statistics and Use, Introduction to Geometry, Indirect Measurement, Signed Numbers, Equations and Problems, Extending Fundamental Operations, Special Products and Factoring, Fractions, Fractions and Literal Equations, Simultaneous Linear Equations, Variation and Proportion, and Demonstrative Geometry.

The evidence gathered by the check list indicates that very little of sections entitled Dependence, Indirect Measurement, Extending the Four Fundamental Operations, Special Products and Factoring, Fractions and Literal Equations, Simultaneous Linear Equations, and Variations and Proportion has a great value in a course in general mathematics designed for usability and therefore should be omitted. The writer recommends the addition of more problems concerning business, insurance, common and decimal fractions, and probably more material on the reading and interpretation of graphs.

This textbook is compiled from materials selected from the fields of arithmetic, algebra, and geometry, the authors apparently attempting to give almost equal space to each of these three fields. It is the writer's opinion that anyone needing as much algebra and geometry as is given in this textbook could probably profit more from the formal algebra and geometry courses than from general mathematics. It is also probable that the majority of students not requiring the traditional algebra and geometry would receive more benefit from the more practical problems which have previously been shown to be of great importance.

3. Practical Mathematics, a textbook by Lennes, is divided into the following sections:

Part I. A Restudy of Grade School Mathematics:
Whole Numbers and Decimals, Common Fractions, Equations, Percentage, Simple Application of Percentage, Graphs, and Measuring.

Part II. Solving Practical Problems:
Cost of Owning a Car; Cost of Owning a House and Other Problems; Insurance; Taxes; Miscellaneous Home Problems; Buying, Preparing, and Serving Food; Household Accounting; Buying Lumber and Problems for Carpenters; Excavating and Cement Work; Useful Geometric Figures; Some Simple Technical Problems; Installment Buying; Farm Problems.

Part III. Quantitative Aspects of Social Problems:
Large Numbers and Their Uses, The Grade Curve, and The March of Progress.

From the results of the check list it appears that the material in Practical Mathematics, by Lennes, is very useful

to many people. Part I and the business problems section of Part II seem to have the greatest value for most people. The results from the check list disclose that even the most elementary of technical problems have a limited use. Hence, it is possible that the section in Lennes' Practical Mathematics embracing the more technical problems could be advantageously omitted. The remainder of the textbook, however, seems to be made up of material which has been very carefully selected and includes the problems which the writer's survey found to be most practical and most frequently used in daily occupational situations.

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APPENDIX

PLEASE STATE YOUR OCCUPATION _____

Please indicate which of the following type problems you have solved since you were graduated from high school.

Write a 2, 1, or 0 in the parenthesis before each item.

Mark those you have solved many times with the numeral two (2).

Mark those you have solved occasionally with the numeral one (1).

Mark those problems you have never solved with a zero (0).

EXAMPLE TO SHOW HOW TO MARK THE PROBLEMS:

Have you solved this type problem since leaving high school:

1. (0) Compound interest
2. (1) Division of decimals
3. (2) Figured area of circle

The person who marked item 1 above with a zero (0) indicates he has never solved a problem of compound interest since leaving high school. Item 2 is marked with a one (1) which shows that this person has used division of decimals occasionally since leaving high school. Item 3 is marked with a two (2) to show that he has figured the area of a circle many times since leaving high school.

ARITHMETIC

Have you since leaving high school:

1. () Used metric system of measurement, as determining length in terms of meters, centimeters, etc.
2. () Measured length of any object and expressed the answer decimally, as 4.4 ft.
3. () Drawn a plan or map to scale
4. () Measured the area of an irregular plot of ground in acres
5. () Figured the number of gallons in a given number of cubic feet, as finding the number of gallons in a tank
6. () Figured the number of bushels of grain in a bin
7. () Figured a lumber bill in which you had to compute the number of board feet
8. () Figured the length of a rafter for a house with a definite pitch
9. () Measured wood by the rank or cord

Fractions and Decimals

10. () Raised fractions to a higher denominator, as $7/10$ is equal to $28/40$.
11. () Added fractions, as $3/16 + 7/8 + 2/3$
12. () Multiplied mixed numbers, as $4\frac{1}{2} \times 5\frac{1}{2}$
13. () Solved problems using addition, subtraction, multiplication, or division of decimals
14. () Reduced mixed numbers to improper fractions, as $3\frac{1}{4}$ to $13/4$
15. () Used table of fractional equivalents as a short method in multiplying, as $12\frac{1}{2} \times 64 = 800$

Percentage

16. () Used per cent to figure interest problems
17. () Used per cent to figure bank discount as in selling an interest bearing note to the bank, then figuring amount after bank had discounted it
18. () Used per cent to interpret graphs, as in bar or circle graphs which use per cent to indicate the ratio of items
19. () Found what per cent one number is of another, as 3 is what per cent of 24
20. () Figured the interest rate when buying on the installment plan
21. () Figured discount given for cash, as the 2% discount often allowed by wholesale houses

Home Expenditures

- 22. () Made out a budget of expenditures over a given period of time, as a budget for household, school, or trip expenses
- 23. () Kept record of money spent in home or business
- 24. () Checked a bill received to see if all items were correct
- 25. () Made out a statement or a bill
- 26. () Made out or signed receipts
- 27. () Read a light, gas, or water meter, and figured the cost of service
- 28. () Figured caloric content in comparing food prices, or planned a meal having correct number of calories

Insurance

- 29. () Figured compound interest, as computed by life insurance or investment companies
- 30. () Investigated life insurance to the extent of learning the different kinds of insurance policies
- 31. () Figured the loan or cash value of an insurance policy

Areas

- 32. () Used the value of π in figuring the area of a circle
- 33. () Found the area of a trapezoid
- 34. () Found the area of a triangle using the rule that the area of a triangle equals $\frac{1}{2}$ the base times height
- 35. () Found the surface of a sphere using the formula $S = 4\pi r^2$

Miscellaneous

- 36. () Checked a personal or property tax receipt to see if correct
- 37. () Figured increase or decrease in taxes using mills as the rate
- 38. () Figured the cost of owning a car, or home, including the original cost, upkeep, interest on investment, taxes, and depreciation
- 39. () Checked a bank balance sheet to see if balance was correct
- 40. () Figured expense of keeping an old car, and compared it with the expense involved in buying a new one
- 41. () Found the volume of a cube
- 42. () Found the square root of a number that has at least three digits, as $\sqrt{624}$
- 43. () Figured the cost of excavating at a given cost per cubic yard
- 44. () Figured the cost of making a concrete wall
- 45. () Figured horse-power needed to pull an object of given weight up a given grade
- 46. () Read or written numbers in the millions or billions

ALGEBRA

- 47. () Drawn or constructed a graph
- 48. () Observed or interpreted graphs in papers, magazines, or books
- 49. () Used median, mode, or average
- 50. () Constructed a graph from a linear equation, as constructing a graph from the equation $F = 1.8 C - 32$
- 51. () Made a graph of a quadratic equation

Factoring

- 52. () Factored a perfect square, as factoring $x^2 - 10x + 25$ into $(x - 5)^2$
- 53. () Removed a common factor from a polynomial, as $3x + 6y - 12z = 3(x + 2y - 4z)$

54. () Factored the sum of two cubes, as $(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$

Fractions

55. () Reduced algebraic fractions to lowest terms, as $\frac{12x^2}{18x} = \frac{2x}{3}$
 56. () Multiplied fractions, as $\frac{-15m^2}{2m} \times \frac{-8n^3}{25m^3} = \frac{12n^3}{5m^2}$
 57. () Changed fractions to equivalent fractions, as $3/7x^2 = 12x/28x^3$
 58. () Simplified a complex fraction, as $\frac{1 + \frac{a-b}{a+b}}{1 - \frac{a-b}{a+b}} = \frac{a}{b}$
 59. () Changed an algebraic fraction in form by changing the sign of the fraction, as $\frac{x}{x-2} = \frac{-x}{x-2}$

Equations

60. () Solved problems representing the unknown with x
 61. () Changed Fahrenheit to Centigrade or visa versa
 62. () Solved equations of the type: $4x - 5 - 2x = 23$
 63. () Solved a simple equation, as $2x = 12$
 64. () Solved fractional equations, as $\frac{x}{7} = 1$
 65. () Cleared fractions and solved equation, as in problem: $\frac{x}{6} - \frac{2x-1}{4} = 1$
 66. () Solved quadratic equation, as $x^2 + 7x + 12 = 0$
 67. () Used formula for solving quadratic equation: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
 68. () Solved a decimal equation, as $.5x + .7x = .48$
 69. () Solved a literal equation, as $ax - b = c$
 70. () Solved a set of linear equations for x and y, as $\begin{cases} x + 2y = 2 \\ 3x + 5y = 7 \end{cases}$

Simple Processes

71. () Added several algebraic numbers, as $5a - a + 12a - 4a$
 72. () Subtracted polynomials, as $(3x - 2y + 7) - (5z + 4y - 8z)$
 73. () Multiplied positive and negative numbers using the rule that plus times plus equals plus, minus times minus equals plus, plus times minus equals minus, and minus times plus equals minus.
 74. () Multiplied monomials, as $a^3 \times a^2 = a^5$
 75. () Multiplied a polynomial by a monomial, as $2x(5x^2 - 8 - 3y^2)$
 76. () Squared a binomial, as $(a - b)^2$
 77. () Divided by long division in algebra, as $2x^3 - 3x^2 - 5x + 6$ by $x^2 - 3x + 2$
 78. () Divided a polynomial by a monomial, as $x^4 - 2x^3 + x^2$ by x^2
 79. () Multiplied monomials together which had negative exponents, as $a^{-2} \times a^{-5}$

Miscellaneous

80. () Used negative numbers
 81. () Used the term exponent
 82. () Solved a problem of this type: One boy can do a piece of work in five days; another can do the same work in seven days. How long will it take to do the work if they work together?
 83. () Solved a mixture problem as: How much water must be added to a quart of 20% solution to reduce it to a 5% solution?
 84. () Found the least common multiple of several expressions, as of $x^2 - xy$ and $3x^2 - 3xy$
 85. () Found the cube root of a monomial expression, as the cube root of $27m^6$

119. () Made two lines parallel to each other by making them perpendicular to the same line
120. () Solved a problem using the principle that angles whose sides are parallel each to each are either equal or supplementary

Proportion

121. () Measured the height of an object using its shadow in comparison with an object of known height and its shadow
122. () Divided a line segment into equal parts by proportion
123. () Found the distance to an inaccessible point by the use of geometry
124. () Solved a problem in proportion in which you used the fact that the product of the means equals the product of the extremes
125. () Solved a problem which made use of the fact that if two quantities are in proportion, they are in proportion by addition or subtraction
126. () Divided a line externally, internally, or harmonically
127. () Added numerators and denominators in a series of equal ratios and set them equal by any one of the ratios
128. () Solved a mean proportion problem in which the tangent to a circle was the mean proportion between the secant and its external segment

Miscellaneous

129. () Used a protractor
130. () Used a square
131. () Observed geometric designs in nature, conscious that they were geometric
132. () Used a drawing board and a T square
133. () Found the area of a triangle using the formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ where s equals $\frac{1}{2}$ the perimeter, and a , b , and c are the sides
134. () Used the word concurrent
135. () Used the term converse
136. () Done landscaping which involved construction of geometric figures
137. () Referred to a variable approaching a limit
138. () Used the type of reasoning employed in geometry: namely, using material given, building up a proof, then drawing conclusion