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## Place Value in Primary Sources OER Activity

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## **Place Value in Primary Sources**

## Dr. Cynthia Huffman, University Professor of Mathematics, Pittsburg State University, Pittsburg, KS (last revised May 2021)

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**Objective:** Examine the use of place-value in the Hindu-Arabic numeration system from a historical viewpoint by looking at primary sources.

**Prerequisites:** Basic familiarity with the Hindu-Arabic base-ten place-value numeration system and Roman numerals I (one), V (five), X (ten), L (fifty), C (100), D (500), M (1000).

Prior to the 13<sup>th</sup> century, Europeans used Roman numerals when they needed to write down numbers. Leonardo of Pisa, also known as Fibonacci, introduced the Hindu-Arabic numeral system to Europe in 1202 in his book *Liber Abacci*. They appear on page 2 at the beginning of the first chapter, where he writes

Incipit primum capitulum. Nouem figure indorum he sunt 9 8 7 6 5 4 3 2 1 Cvm his itaque nouem figuris, et cum hoc signo 0, quod arabice zephirum appellatur, scribitur quilibet numerus, ut inferius demonstratur. Nam numerus est unitatum per-

Translated into English:

These are the nine figures of the Indians

9 8 7 6 5 4 3 2 1

And so, together with these nine figures, and with this symbol 0, which is called zephyr by the Arabs, any number can be written, which is demonstrated below.

Fibonacci then goes on to explain place value, since this is a new concept to someone only familiar with the Roman numeration system, which is mostly additive. His explanation includes discussing the value of the different positions from right to left: units or ones, tens, hundreds, thousands, and on "in infinitum". Following the place value explanation, there is a table containing examples of Roman numerals written with Hindu-Arabic numerals.



M .I MMXXIII		MMMxx	MMMMMdc			Mccxxxiiii	MMMMcccxx
1001	3022			3000	1111		

TASK 1: A. Fill in the missing blanks in the table below from Fibonacci's *Liber Abbaci*.

**B.** 1. How many digits are used for each of the Hindu-Arabic numerals in the second row of the above table?

2. What about the corresponding Roman numerals in the first row?

**3.** What do you conclude about the length of the written numerals in the examples from the table (for example, MI and 1001 or MMMMCCCXXI and 4321) in the two different numeration systems?



Later authors of arithmetic books followed Fibonacci's lead, starting with an explanation of the Hindu-Arabic place value system and then providing several examples. The picture below is from an Italian book called *Libro d'Abaco* by Tagliente, first published in 1515.

Figure di la pratica Nume Regina & fundamento do Vno. dua.tre. qtro.ciq.sfei.sett	gniarte.
·I· ·2· ·3· ·4· ·5· ·6·	.789
Prima figura fignifica nume	
Secunda figura fignifica nume Terria figura fignifica nume	ro de centenara
Quarta figura fignifica nume Quinta figura fignifica nume	ro de.x.de mille
Sexta figura fignifica nume Sentima figura fignifica nume	ro de.c. de mill.
Ortaua figura fignifica nume	ro de.x.de munon
Decima figura fignifica num	ero de.c.de mul.
DES ALESSA	

Next Tagliente has a full page of examples of numbers written using the Hindu-Arabic numeral system. There are two columns. The left column contains the Hindu-Arabic numerals and the right column has the numbers written out in Italian.

**TASK 2: A.** Complete the middle column of the following table. On the left are 24 Hindu-Arabic numerals in the order they were given on leaf 5. (Even by the early 16<sup>th</sup> century, not all books had page numbers.) The right column has the Italian number names scrambled. Fill in the middle column with the correct Hindu-Arabic numeral, from the choices on the left, to match the corresponding Italian name on the left. The table continues on the next page.

Possibilities to use in the middle column	Fill in the correct Hindu-Arabic numeral to match the name on the right	Numerals in Italian
10		cento
14		cig millia. 8. ceto. 89
16		diexe
34		diexe millia
77		doe millia treseto. \$7.
99		ducento trenta vno
100		4.miliio.958.ml.360
231		475.millio.789.mi.124.
391		mille
658		nonanta noue
999		noue cêto nonâta .9.
1000		noue mi. 9. cento. 99.
2357		18.millia trecento.2.
3179		otto millia, 7. ceto, 7.
5889		quatordexe
6333		fedexe
8707		sei ceto cinquata otte
9999		fetanta lette
10000		fie millia trecento.33.
18302		347.millia.916.
347916		trea millia cento.79.
4958360		trecento nonanta vne

23895438	trenta quatro
475789124	23.mill.ö.895.mi 438

**B.** Did you notice any patterns? Abbreviations? What does the word "mille" or "millia" mean? Any questions or comments?



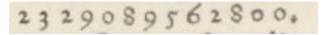
You may have noticed that a tilde ~ was sometimes put over a vowel in the Italian words above as an abbreviation for the letter "n" to follow. For example, *ceto* was used for *cento* (for 100), *treceto* was used for *trecento* (for 300), and *millio* for *million*.

Although Tagliente used the word million as we do today for  $1,000,000 = 10^6$ , not everyone in the  $16^{\text{th}}$  century did. Below are images from *Arithmetica Integra* by Michael Stifel, a friend of Martin Luther. This work was published in 1544 in Latin. He starts out on page 1 mentioning the ten digits.

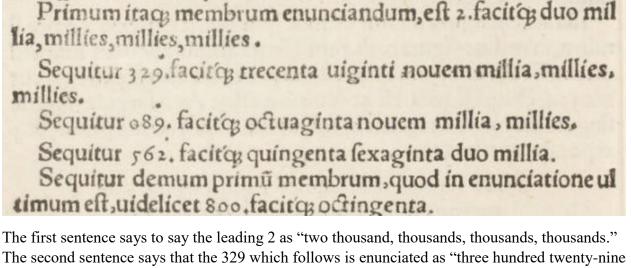


IGVRIS decem utimur, tanquam neceffarijs ad numerorum repræsentationem, uidelicet 1.2.3.4. 5.6.7.8.9.0. Et nouem quidem priores, fignificati uæ uocantur, Quælibet enim earum fignificat nus merum sui ordinis, sicut eas uides esse positas: ut 7.

Then he gives a brief explanation of place value using the example 324, before the following example.



Stifel explains that while the 2 is the last of the 13 digits (figura), it is the first when it comes to saying the number out loud. Then he goes on to give the entire enunciation of the number.



The second sentence says that the 329 which follows is enunciated as "three hundred twenty-nine thousand, thousands, thousands." For the third sentence, the 089 which comes next is read as "eighty-nine thousand, thousands." The next sentence is left as part of Task 3. The last sentence says that finally the first member follows, which is read last, is clearly 800, and is "eight hundred."



Task 3: A. Translate the fourth sentence from the Stifel excerpt above.

B. Write out in words how you would enunciate Stifel's 13 digit example: 2,329,089,562,800

# **KEY**

### TASK 1: A.

M .1	MMxxIII	MMMxxII	MMMxx	MMMMMdc	MMM	Mcxi	Mccxxxiiii	MMMMcccxxi
1001	2023	3022	3020	5600	3000	1111	1234	4321

### В.

- 1. All the Hindu-Arabic numerals in the second row of the table have 4 digits.
- 2. The number of symbols used in the Roman numerals varies.
- **3.** One advantage of the Hindu-Arabic system of numeration over the Roman system is that one can tell the relative "size" of a number by how many digits are used.

### **TASK 2:**

**A.** 100, 5889, 10, 10000, 2357, 231, 4958360, 475789124, 1000, 99, 999, 9999, 18302, 8707, 14, 16, 658, 77, 6333, 347916, 3179, 391, 34, 23895438

**B.** Answers will vary – might include "mille = 1000" and "~" for "n".

### **TASK 3:**

A. 562 thousand

**B.** two trillion, 329 billion, 89 million, 562 thousand, 800