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### Cross Ratio

Peyton Burlingame

*Pittsburg State University*

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# Cross Ratio

Peyton Burlingame

Pittsburg State University

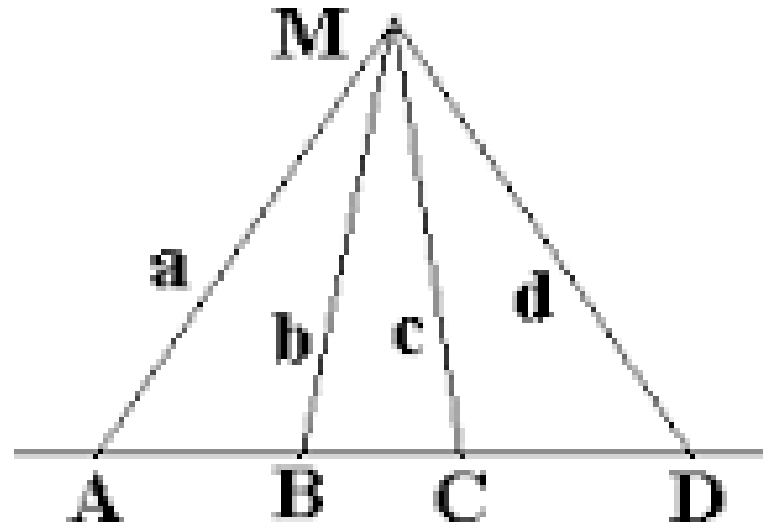
# Problem

Given four points  $A$ ,  $B$ ,  $C$ , and  $D$  in order on a line in Euclidean space, under what conditions will there be a point  $P$  off the line such that the angles  $\angle APB$ ,  $\angle BPC$ , and  $\angle CPD$  have equal measure.

-MAA Monthly Problem #11915

# Cross Ratio

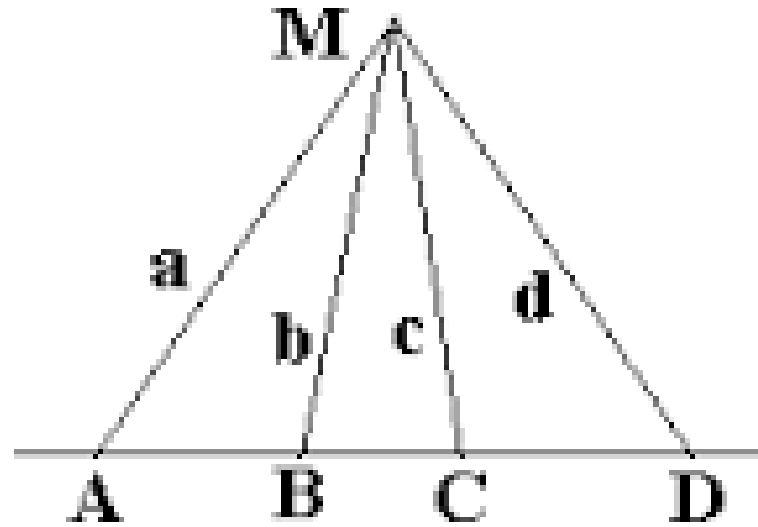
$$(ABCD) = \frac{\frac{AC}{CB}}{\frac{AD}{DB}}$$



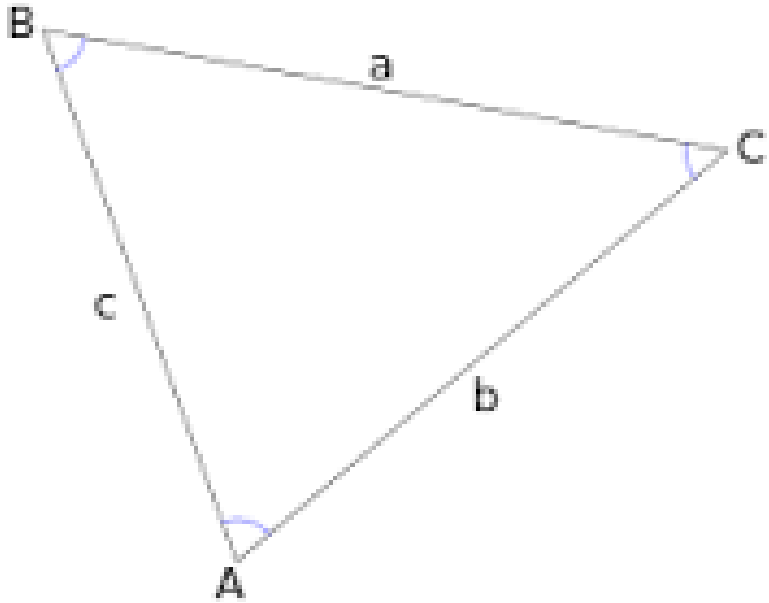
Geogebra Link

# Cross Ratio Theorem

The cross ratio will be the same for any line crossing the four rays ( $a, b, c, d$ ) starting at point  $M$ .



# Law of Sines

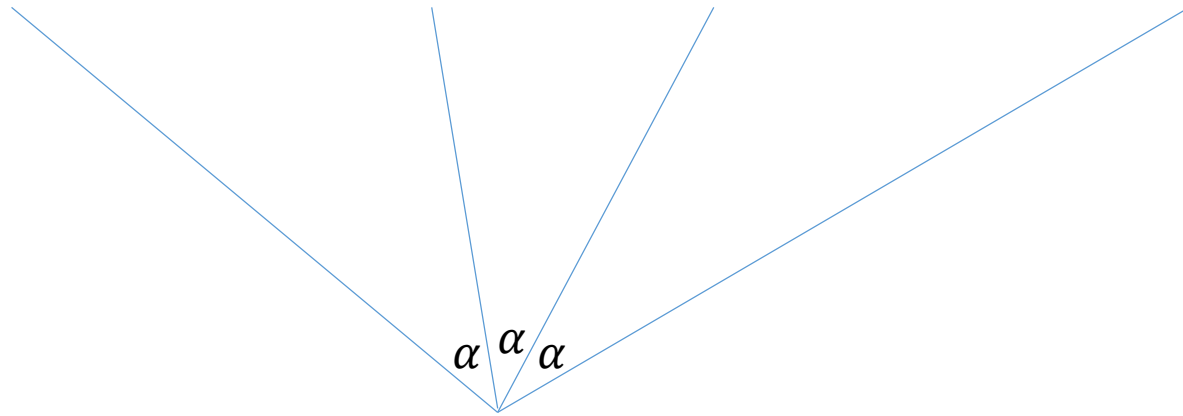


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{\sin A}{\sin B} = \frac{a}{b}$$

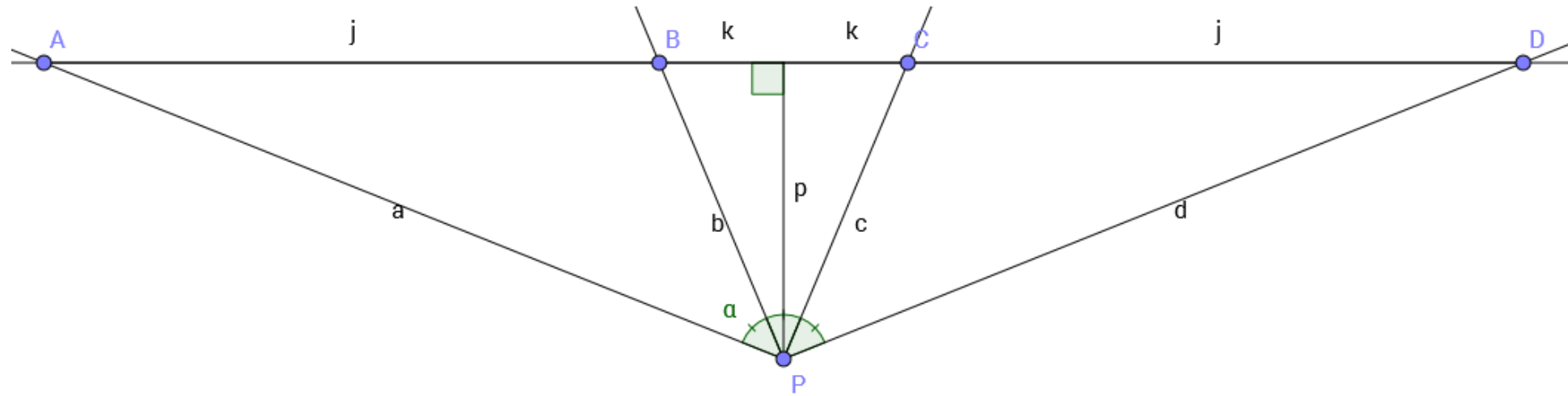
# Cross Ratio Property

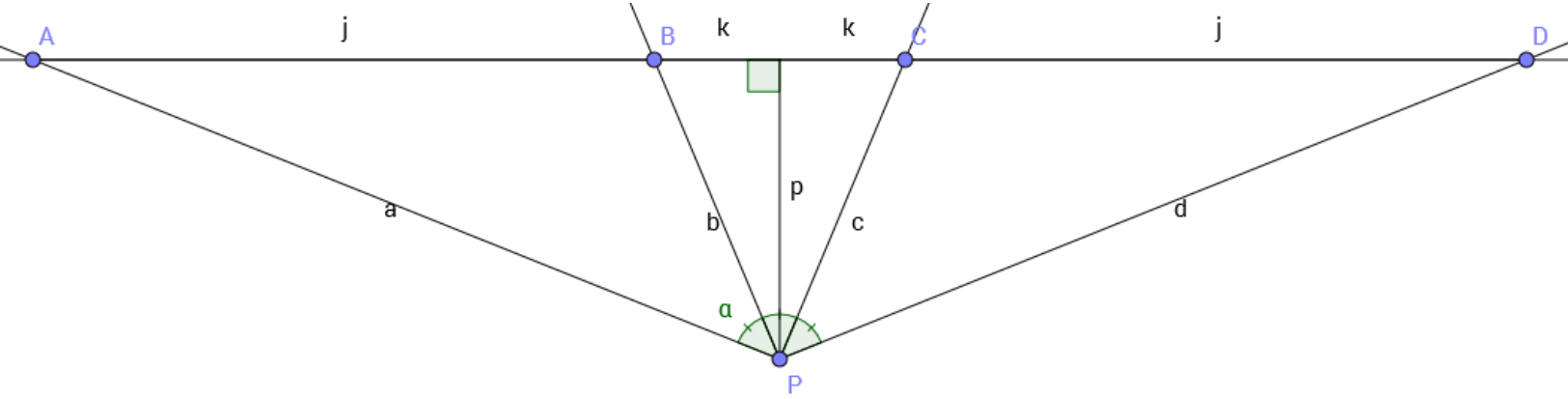
If the three angles are congruent, the cross ratio must be greater than  $4/3$ .



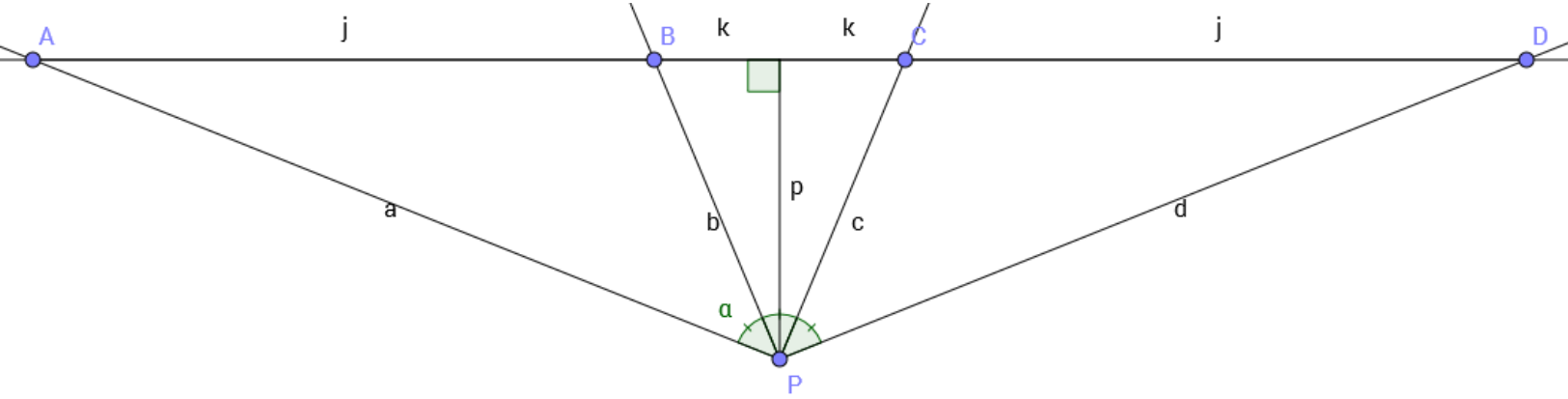


# Best situation





$$(ABCD) = \frac{\frac{AC}{CB}}{\frac{AD}{DB}} = \frac{\frac{j+2k}{2k}}{\frac{2j+2k}{j+2k}} = \frac{(j+2k)^2}{2k(2j+2k)}$$



$$(ABCD) = \frac{(j + 2k)^2}{2k(2j + 2k)} = \frac{\left[ \tan\left(\frac{3}{2}\alpha\right) + \tan\left(\frac{1}{2}\alpha\right) \right]^2}{4 \tan\left(\frac{1}{2}\alpha\right) \times \tan\left(\frac{3}{2}\alpha\right)}$$

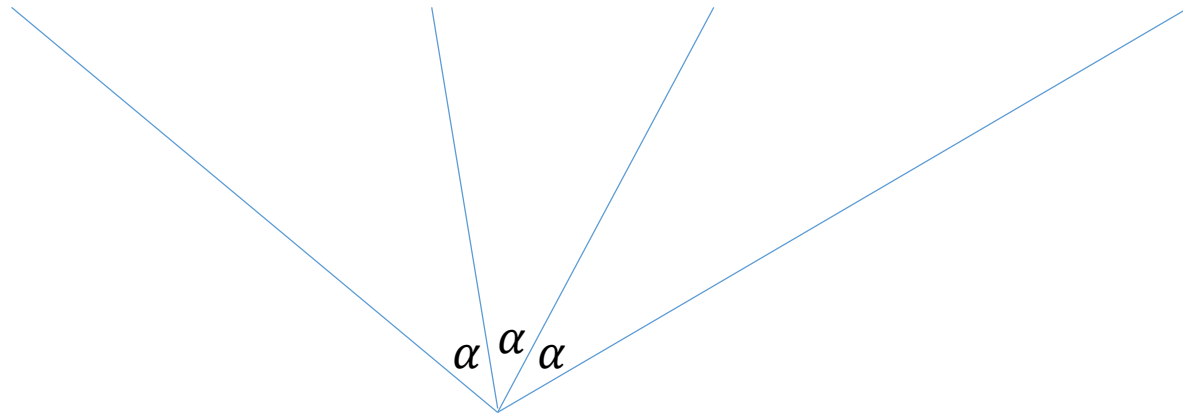
# Cross Ratio is Increasing

On the interval  $\left(0, \frac{\pi}{3}\right)$ ,

$$\frac{d}{d\alpha} \left( \frac{1}{2 \cos(2\alpha)+1} + 1 \right) = \frac{4 \sin(2\alpha)}{(2 \cos(2\alpha)+1)^2} > 0$$

# Cross Ratio Property

If the three angles are congruent, the cross ratio must be greater than  $4/3$ .



# Problem

Given four points  $A$ ,  $B$ ,  $C$ , and  $D$  in order on a line in Euclidean space, under what conditions will there be a point  $P$  off the line such that the angles  $\angle APB$ ,  $\angle BPC$ , and  $\angle CPD$  have equal measure.

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# Partial Solution

If there exists a  $P$  then the cross ratio is greater than  $\frac{4}{3}$ .

Areas of further work are necessary



# Acknowledgments

- Dr. Leah Childers
- Appreciation to the Honors College for encouraging us to do this project

# References

- Geogebra
- Wolfram Alpha
- Milne, John J. *An Elementary Treatise on Cross-ratio Geometry, with Historical Notes*. Cambridge: U, 1911. Print.