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What is the Shape of a traditional Rapanui house on Easter Island? A Multicultural Mathematical Activity Involving Ellipses

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A Multicultural Mathematical Activity Involving Ellipses

by

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The Rapanui people of the island Rapa Nui, also known as Easter Island, at one time had a system of “writing” called Rongorongo. Unfortunately, the ability to read Rongorongo has since been lost. So currently archaeology and oral tradition are the only available sources for information about the early people of the island. Oral tradition states that the houses originally had upside down canoes for their roofs, and thus they are called *hare paenga*, or in English, boat-houses. In the literature, these boat-houses are said to be elliptical in shape. In this activity, we will investigate the shape of archaeological remains of foundations of the boat-houses to try and determine if they are indeed in the shape of an ellipse.

Dr. Cynthia Huffman at the moai quarry Rano Raraku on Easter Island (July 2019).

Picture of the foundation of a hare paenga (boat house) on Easter Island (photo by Dr. Cynthia Huffman, July 2019).

Review of Ellipses:

For simplicity, let us assume that our ellipse is set up on a coordinate system, centered at the origin, with the major axis in the vertical or y direction, and the minor axis in the horizontal or x direction. One form for the equation of a generic ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $2a$ is the length of the minor axis and $2b$ is the length of the major axis.

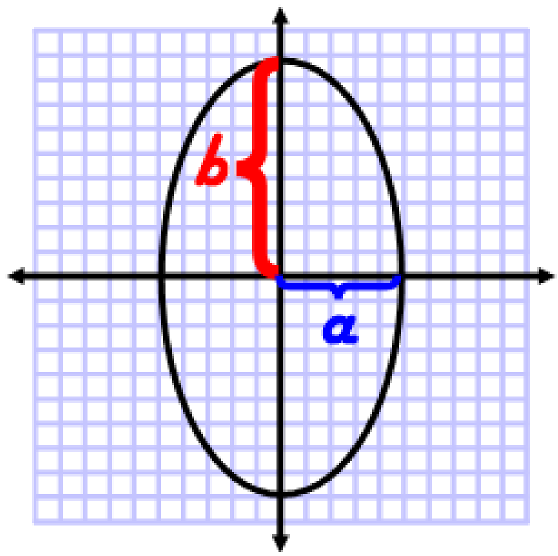

Figure 1

An ellipse is often defined as a planar shape consisting of the set of all points whose distances from two fixed points, called foci or focuses, add up to a constant sum. With an ellipse oriented as above, with the major axis in the vertical direction, the foci would be located on the y-axis with each focus the same distance from the center of the ellipse. In the graph below, the distance from a focus to the origin is labelled c and the foci F_1 and F_2.

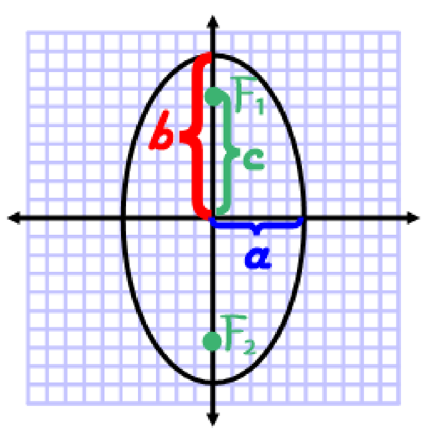

Figure 2

As mentioned earlier, if P is any point on the ellipse, then the sum of the distance from P to F_1 added to the distance P to F_2 is a constant.

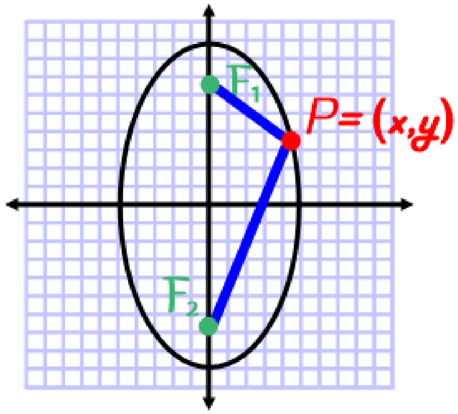

Figure 3

We can determine this constant sum by considering a particular point, namely one of the endpoints of the major axis, say $(0,b)$.

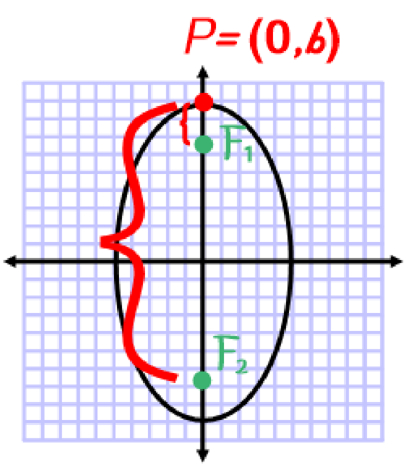

Figure 4

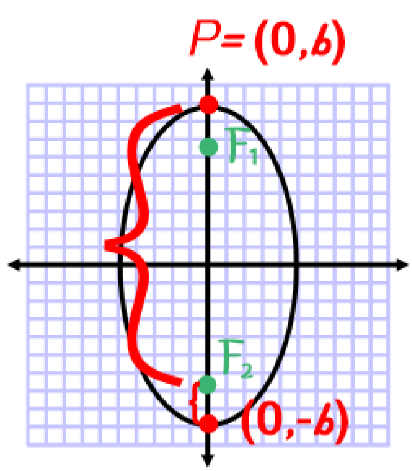

Figure 5

So the constant we are trying to determine is equal to the sum of the distance from $(0,b)$ to F_1 plus the distance from $(0,b)$ to F_2 (see Figure 4). Then, using the symmetry of the foci about the origin, the constant is also the sum of the distance from $(0,-b)$ to F_2 plus the distance from $(0,b)$ to F_2 (see Figure 5), which is the length of the major axis, $2b$ (see Figure 1).

Now, that we know the sum of the distances from any point on the ellipse to the foci is $2b$, then it is also true for the particular point $(a,0)$ (see Figure 6). So, using the symmetry of the ellipse, we have that the distance from a focus to $(a,0)$ is b.

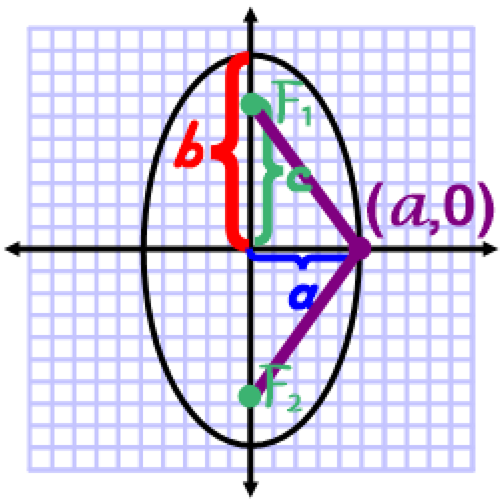

Figure 6

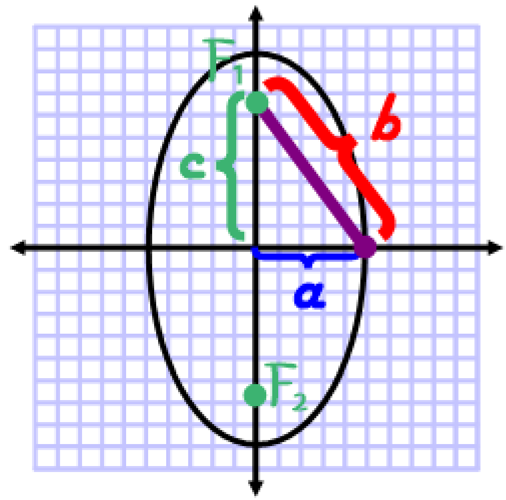

Figure 7

Consequently, by using the Pythagorean Theorem, we find that $c^2 + a^2 = b^2$ and $c = \sqrt{b^2 - a^2}$.

Thus, the general ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, with major axis in the vertical direction, has major axis of length $2b$, minor axis of length $2a$, and the distance from a focus to the center of the ellipse is $c = \sqrt{b^2 - a^2}$.

Application of Ellipses to the Rapanui Boat-Houses (Hare Paenga):

If the shape of the Rapanui boat-houses were ellipses, then the foundation could have been laid out in the following way. Set two poles in the ground along the desired longer axis of the house, spaced evenly from the center of the house. Tie the ends of a rope to each pole so that the length of the rope when pulled taut is equal to the desired length of the house. Using a short pole or tool with a sharp end, pull the rope taut and trace out an ellipse on the ground.

To algebraically describe a boat-house foundation, one could measure across the widest spots in each of the two perpendicular directions of the foundation, and then substitute these lengths for a

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and b using the form of the equation of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. For example, if the length of a foundation was 26 feet and the width 10 feet, then the equation would be $\frac{x^2}{5^2}+\frac{y^2}{13^2}=1$ or $\frac{x^2}{25}+\frac{y^2}{169}=1$. If we let c be the distance from the center of the ellipse (in our case the origin) to a focus of the ellipse, then $c=\sqrt{b^2-a^2}$. Since we are letting b be the major axis, b^2-a^2 will be a positive value. For our example, $c^2=b^2-a^2=13^2-5^2=169-25=144$. So, c is 12 feet. Thus, the foundation could have been laid out using poles that were set $2c=24$ feet apart, using a rope that, after being tied to the two poles, had length $2b=26$ feet. So, the rope when pulled taut along the axis of the two poles, would have extended 1 foot past the nearest pole.

Outdoor Activity: Laying out a Rapanui Boat House Foundation

This activity can be done together in a class or in groups of 4 to 6 students. A sand volleyball pit would be an ideal location. A parking lot or another outside flat area would also work.

Supplies needed:

- 2 sturdy poles (the length isn't important; 4 feet works well)
- Rope (the length will roughly determine the length of the foundation, clothesline rope is a possibility)
- Chalk (if on a parking lot or paved surface) or a stick (if on the ground or a sand volleyball pit) for tracing the ellipse

1. Have 2 students hold the poles vertically. These will be the foci of the ellipse. They should be at a distance of at least 3 feet less than the length of the rope.
2. Tie each end of the rope to one of the poles. There should be some slack left in the rope after it is tied to the poles.
3. While students are holding the poles firmly, another student uses the chalk or stick to pull the rope taut and to mark out the ellipse while walking around the poles. (When near the poles, the student will need to stop and move around the pole to be able to continue.)

Indoor Variation:

The above activity can be modified to be done on a smaller scale indoors by students in groups of 2 or 3 using toothpicks, string, and tracing the ellipse on paper with a pencil.