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CGI (Computer Generated Imagery) Matrix Multiplication Activity

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CGI Matrix Multiplication Activity for High School or College Algebra

by

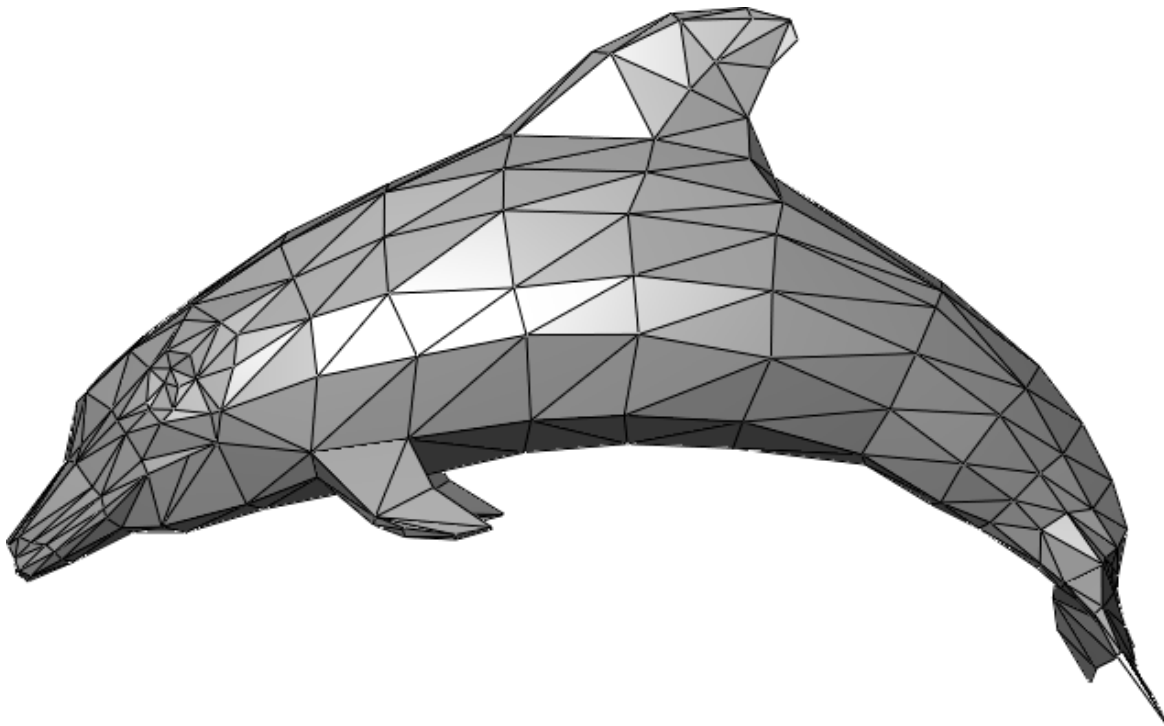
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Many students are motivated by real world applications of mathematics, and most students are familiar with the term CGI from watching movies and playing video games. This particular activity ties matrix multiplication to CGI (Computer Generated Imagery). It has been used successfully with high school students in an Algebra II course who had just learned how to do matrix multiplication, middle school and high school students (some of whom were seeing matrix multiplication for the first time) attending a special Math Day on a university campus, and with College Algebra students (during a unit about matrices).

One way that objects are modeled mathematically in CGI is with a polygon mesh, consisting of vertices, edges, and faces, as in the figure of the dolphin below. The more polygons used, the more realistic the object, but also the more expense/time/work involved in animating the object in a movie or video game.

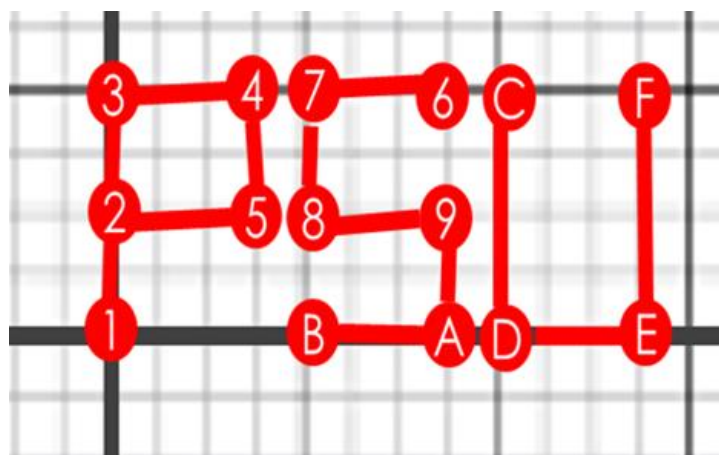


*By en:User:Chrschn - http://en.wikipedia.org/wiki/File:Dolphin_triangle_mesh.png, Public Domain,
<https://commons.wikimedia.org/w/index.php?curid=10626667>*

In order to animate the object, transformations are applied to the vertices, which are points. To distinguish them from vectors, the two-dimensional points are written in what are called homogenous coordinates; that is, the point (x, y) in the plane is written as $(x, y, 1)$. (The vector $\overline{(x, y)}$ would be written as $(x, y, 0)$.) All linear transformations, and many other transformations such as translations, can be applied by using matrix multiplication. So, if the point is written as a column matrix, the students can perform CGI-like transformations to a point by hand, by multiplying a 3×3 - matrix times a 3×1 - matrix, just like what is being done on a larger scale by CGI animators using computers to apply transformations to vertices in a polygon mesh.

Preparing for the Activity

1. Decide what object you are going to “animate” and lay it out on a coordinate system to assign points to the students. It’s helpful if the number of points is the same as the number of students. If the number of points is fewer than the number of students, you can have students work in groups, or have duplicates. If the number of points is more than the number of students, some of the students may need to do the matrix multiplication computations with more than one point. For example, I have used the initials of the students’ school, as in the figure below.



2. Have ready some way to let each student know their assigned point. I put the coordinates with point labels in a large font in a document, which I print off and then cut into pieces to hand out to the students, like in the figure below.



1 (0,0)	2 (0,2)	3 (0,4)	4 (3,4)
5 (3,2)	6 (7,4)	7 (4,4)	8 (4,2)
9 (7,2)	A (7,0)	B (4,0)	C (8,4)
D (8,0)	E (11,0)	F (11,4)	

- Plan how you will have the students plot points. Possibilities include having a coordinate system drawn on a whiteboard or chalkboard, and letting the students first plot their original points (all in one color), followed by their transformed points in another color. Or, a coordinate system could be projected on a screen and students plot points using sticky notes.
- Have handouts with the transformation matrices ready, like the one at the end of this article, or have the matrices displayed for the students to see.

Steps in the Activity

- Introduce the activity. I have used something similar to the following. Ask the students if they have ever seen a movie or played a video game, and if they know what CGI is. Show an example of a polygon mesh (like the one on the first page or one from an image search of “polygon mesh” and the name of a favorite movie character or movie company). Mention how a character or object can be animated by applying transformations to the points that make up the vertices of the polygon mesh, and that many of these transformations are carried out by matrix multiplication.
- Pass out the matrix handouts and the points to the students. Explain that the two coordinates of their point (x, y) are to be placed in the column matrix so that it reads $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$.
- Have the students do the first matrix multiplication and plot their results. Discuss the transformation and how it relates to the matrix. If you are using the attached handout, the first matrix results in a translation 2 units in the positive x direction and 6 units in the positive y direction. Many students will be able to notice the connection with the 2 in the first row of the matrix and the 6 in the second row.



4. Continue with each matrix, one at a time, having students plot their transformed points and then discuss the resulting transformation and the associated matrix. If you use the attached handout, the second matrix results in a 90° counter-clockwise rotation, the third matrix results in a 90° counter-clockwise rotation followed by a translation of 5 units in the positive x direction, the fourth matrix results in scaling by 2 in the horizontal direction and 5 in the vertical direction (a real-world example of scaling a different amount in each of the horizontal and vertical directions is what happens the moment a soft rubber ball hits the floor), and the fifth matrix is a rotation and a shear and requires a larger coordinate system than the other transformations. (Shadows are real-world examples of this type of transformation.)

Reference

Introduction to the Mathematics of Computer Graphics, Nathan Carter, MAA Classroom Resource Materials, Volume: 51; 2016, <https://bookstore.ams.org/clrm-51/>



Matrices and CGI handout

Using your point, perform the following matrix multiplications.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 6 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -1 & 5 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \\ 1 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \\ \\ 1 \end{bmatrix} =$$