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Maria Agnesi Activity

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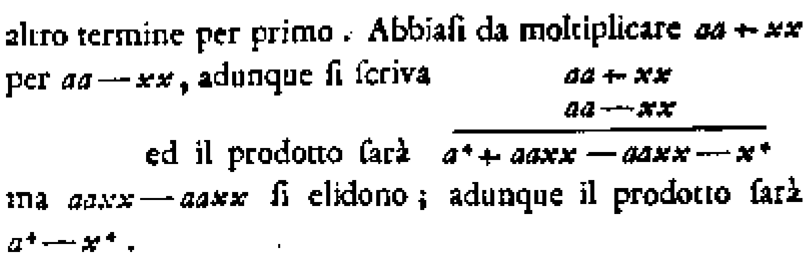
Overview: This activity was originally created for a Women in Mathematics course to provide students with a small taste of some basic mathematics connected to the work of Maria Agnesi. The activity has the students look at some of her work in algebra and calculus (optional). It could also be used in other courses, such as history of math, a general education mathematics course, high school or college algebra, and calculus.

Public Domain, https://commons.wikimedia.org/wiki/File:Maria_Gaetana_Agnesi.jpg

1. The image below is from Agnesi's *Analytical Institutions*.

a. Looking at the mathematical notation (and not worrying about the words being in Italian), write the problem in modern notation.



altro termine per primo. Abbiaſi da moltiplicare $aa + xx$ per $aa - xx$, adunque ſi ſcriva

$$\begin{array}{r} aa + xx \\ aa - xx \\ \hline \end{array}$$

ed il prodotto ſarà $a^4 + aaxx - aaxx - x^4$

ma $aaxx - aaxx$ ſi elidono; adunque il prodotto ſarà $a^4 - x^4$.

b. What is a translation of the Italian word "elidono"? (Feel free to use a translation app or program, such as Google Translate.) Does this fit what is happening with the mathematics? What do you think would be an appropriate translation?

c. John Colson was an Englishman who learned Italian so that he could translate Agnesi's work *Analytical Institutions* into English. Below is the same selection, but from Colson's English translation of Agnesi. How did Colson translate "elidono"? What do you think of his translation?

$$\begin{array}{l} aa + xx \\ aa - xx \\ \hline a^4 + a^2x^2 \\ \quad - a^2x^2 - x^4 \end{array}$$

from the right hand or from the left, in regard to either of the factors; or which of them is wrote above, and the other below; or in what order the terms are placed. Suppoſe we were to multiply $aa + xx$ by $aa - xx$; proceed as in the margin, where becauſe $aaxx$ and $-aaxx$ deſtroy each other, the product will be reduced to $a^4 - x^4$.

2. The image at the top of the next page is Colson's translation of Agnesi's instructions for solving certain quadratic equations. Apply Agnesi's formula to solve the following quadratic equations. Check that the values are solutions of the given quadratic, either by plugging back in or by re-solving using the modern quadratic formula.

a. $x^2 + 2x = 8$

c. $x^2 + 6x = 4$

b. $x^2 + 3x = 4$

d. $x^2 + x = 5$

74. But if the equation contain the unknown quantity raised to it's square, together with the rectangle or product of the same into known quantities, which is called the second term (and such an equation is called an *Affected Quadratick*, as it is called a *Simple Quadratick* when this second term is wanting); this being prepared as is aforesaid, to both members of the equation must be added the square of half the co-efficient of the second term, (that is to say, the square of half that quantity, whether integer or fraction, by which the unknown quantity is multiplied,) and then it is plain that the first member will always be a square, the root of which will be the aggregate of the unknown quantity, and of the half co-efficient with it's proper sign. And then extracting the root, this aggregate shall be equal to the square-root of the other member of the equation; and transposing the half co-efficient as a known quantity, we shall finally have the unknown quantity equal to the sum or difference (according to the nature of the signs) of the radical and the said half co-efficient. Thus let the equation be $xx + 2ax = bb$: if we add to each member the square of half the co-efficient of the second term, that is aa, the equation will be $xx + 2ax + aa = aa + bb$, and extracting the square-root, it will be $x + a = \pm\sqrt{aa + bb}$, and by transposing, it is $x = \pm\sqrt{aa + bb} - a$. Affected quadraticks resolved.

3. Look again at how Agnesi derived her formula for solving a quadratic of the form $x^2 + 2ax = b^2$ in the image above. Then solve this quadratic by the method of completing the square. What do you notice?

4. (Requires Calculus I knowledge.) At the time of Maria Agnesi, the definition of a function had not yet been formalized. So, people often worked with curves in the plane rather than functions as we do toady. When one needed a derivative for an application such as finding a tangent line or determining a maximum or minimum, implicit differentiation would be used. Below is a problem from page 60 of Volume 2 of Colson's translation. Notice he is using a dot above a variable to mean its derivative, which is Newton's notation, and he talks about fluxions rather than derivatives or differentials. Why would Colson be using Newton's notation? Do you think the Agnesi's original Italian used the same notation? (To find out, see the images on the next page.) Restate and solve the problem as you would expect it to appear in a Calculus textbook today.

EXAMPLE I.

75. Let there be a curve with this equation $2ax - xx = yy$, and let it be required to know, to what point of the axis, or of the abſciſs x, the greateſt ordinate y correſponds, and what that ordinate is.

The fluxional equation of this will be $2a\dot{x} - 2x\dot{x} = 2y\dot{y}$, that is, $\frac{\dot{y}}{\dot{x}} = \frac{a-x}{y}$. Making the ſuppoſition of $\dot{y} = 0$, the numerator of the fraction ought to be nothing, or $a - x = 0$, whence $x = a$. Therefore the greateſt ordinate belongs to that abſciſs which is equal to a. This value being ſubſtituted inſtead of x in the propoſed equation, it will be $2aa - aa = yy$, that is, $y = \pm a$. Therefore the greateſt ordinate, poſitive and negative, will be equal to a. Making the ſuppoſition of $\dot{y} = \infty$, the denominator of the fraction ought to be nothing, and therefore it will be $y = 0$. Wherefore, ſubſtituting this value inſtead of y in the propoſed equation, we ſhall have $x = 0$, and $x = 2a$; which is as much as to ſay, that $x = 0$ will be the leaſt, and $x = 2a$ the greateſt: Or, more properly, that, when $x = 0$, and $x = 2a$, then $\dot{y}$ being infinite in reſpect of $\dot{x}$, the ſubtangent will be nothing, or the tangent will be parallel to the ordinate y.

ESEMPIO I.

75. Sia la curva dell' equazione $2ax - xx = yy$, e si voglia sapere, a quale punto dell' asse dell' assisse x corrisponda la massima ordinata y, e cosa ella sia.

Differenziata l'equazione, sarà $2adx - 2xdx = 2ydy$, cioè $\frac{dy}{dx} = \frac{a - x}{y}$. Facendo la supposizione di $dy = 0$, dovrà essere zero il numeratore della frazione, e però sarà $a - x = 0$, onde $x = a$; adunque la massima ordinata corrisponde a quell'assissa, che sia eguale ad a; sostituito questo valore in luogo di x nella proposta equazione, sarà $2aa - aa = yy$, cioè $y = \pm a$; la massima ordinata adunque positiva, e negativa è eguale ad a. Facendo la supposizione di $dy = \infty$, dovrà essere zero il denominatore della frazione, e però sarà $y = 0$, sosti-

sostituito per tanto questo valore in luogo di y nella proposta equazione, averemo $x = 0$, ed $x = 2a$; vale a dire, che $x = 0$ sarà la minima, ed $x = 2a$ la massima; o più propriamente, che quando sia $x = 0$, ed $x = 2a$, essendo infinita la dy rispetto alla dx, la sottotangente sarà nulla, cioè la tangente parallela alle ordinate y.
