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### Maryam Mirzakhani Riemann Surface Activity

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# Maryam Mirzakhani Riemann Surface Activity

Dr. Cynthia Huffman, Pittsburg State University

**Overview:** This activity was originally created for a Women in Mathematics course to provide students (who may not yet have taken much upper level mathematics) with a small taste of some basic mathematics connected to the work of Maryam Mirzakhani. In this activity, students investigate some Riemann surfaces and one non-example. The activity could also be used in other courses, such as a general education mathematics course, a course for pre-service elementary teachers, or a history of mathematics course.



[https://commons.wikimedia.org/wiki/File:Maryam\\_Mirzakhani\\_in\\_Seoul\\_2014.jpg](https://commons.wikimedia.org/wiki/File:Maryam_Mirzakhani_in_Seoul_2014.jpg)

In her short life, Maryam Mirzakhani was able to prove many deep and important results. The citation for her Fields Medal states that it was “for her outstanding contributions to the dynamics and geometry of Riemann surfaces and their moduli spaces.” One of the things that she did was to generalize a result known for surfaces of genus 2 or less to surfaces of any genus. A sphere is a Riemann surface of genus zero, while a torus is one of genus 1, a 2-torus is of genus 2, etc...



Sphere



Torus



2-Torus

A Riemann surface is a surface that if you zoom in at a point (i.e., view it locally), it looks like the complex plane. You can think of it like a ball (sphere) which has holes and/or handles added to it. In this activity, we will construct a couple of Riemann surfaces using paper as well as a surface which is not Riemann. After you have completed the constructions take a picture (or pictures) of your results to submit.

## Cylinder

An open cylinder can be viewed as a sphere which has 2 punctures in it, so it is a Riemann surface. The diagram below is one way to represent a cylinder if we think of identifying the two arrows or gluing together the two arrows.



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**Construction 1:** Cut out a rectangle from a piece of paper and tape together one pair of opposite edges to form an open cylinder.

## Möbius Strip

A Möbius strip is not a Riemann surface because it is not orientable. The diagram below is one way to represent a Möbius strip. It is important to identify\glue the arrows so that they are going the same direction.



**Construction 2:** Cut out a rectangle from a piece of paper and tape together one pair of opposite edges after putting in a half-twist (so that the arrows are going the same direction) to form a Möbius strip.

**Investigation:** Start at a point in the middle of the Möbius strip and trace a line along in the middle of the strip until you get back to the point where you started. What do you notice? Does the same thing happen with the cylinder that you made? To see that the Möbius strip is non-orientable, at some point draw an arrow from the middle pointing to the edge on the right. Now imagine you are a bug on the Möbius strip and “walk” along your middle path until you get back to where you started. What do you notice about the direction that your arrow is pointing?

**Optional Extension 1:** For a cute related video, I highly recommend watching the Vi Hart video *Wind and Mr. Ug*. <https://youtu.be/4mdEsouIXGM>

**Optional Extension 2:** Visit <https://www.ctd.northwestern.edu/blog/rainy-day-activity-make-m%C3%B6bius-strip> for more fun activities with Möbius strips.

## Torus

A torus, or informally, a doughnut, is a Riemann surface of genus 1. It can be formed by attaching one handle to a sphere. The diagram below is one way to represent a torus if we think of identifying or gluing together the arrows of the same color in the same direction.



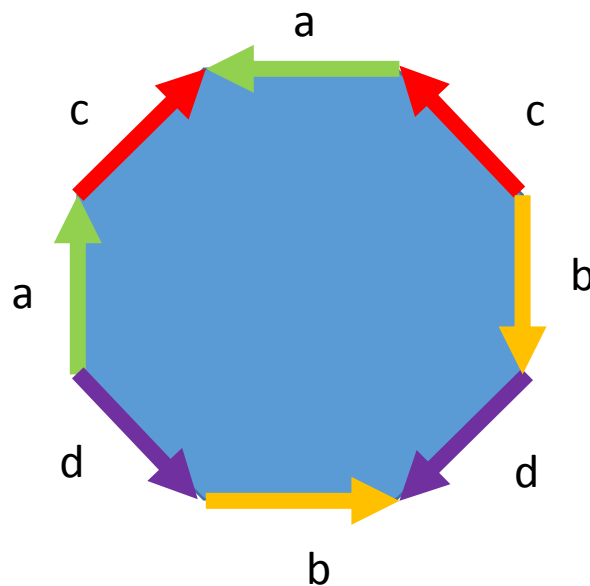
**Construction 3:** Cut out a rectangle from a piece of paper. It will work better if the length is at least 7 times the width. You may need to “soften” up the paper to make it more flexible by wrapping it around a pencil or finger. Next tape together the two longer sides, forming a long cylinder. Then swing around the two ends and tape them together. To see a Mathematica animation of the construction, visit

<https://mathematica.stackexchange.com/questions/42493/morphing-a-sheet-of-paper-into-a-torus/42593> and scroll down a bit.

**Optional Extension:** If you crochet, knit, or sew, you could make a torus. Feel free to try just using the diagram with arrows, or you can find patterns and instructions by searching on the web.

## Double Torus or 2-Holed Torus

A double torus is a Riemann surface of genus 2. It can be formed by attaching two handles to a sphere. The diagram below is one way to represent a torus if we identify or glue together the arrows of the same color in the same direction.



Watch <https://youtu.be/G1yyfPShgqw> to see an animation of the double torus being formed. A crocheted double torus can be seen at <https://qulog.wordpress.com/2007/03/21/double-torus/>.

**Optional Extension:** Create a double torus out of paper or some other material.