7-28-2018

Ada Byron Lovelace Activity

Cynthia J. Huffman Ph.D.
Pittsburg State University, cjhuffman@pittstate.edu

Ada Byron Lovelace Activity

Dr. Cynthia Huffman, Pittsburg State University

Overview: This activity was originally created for a Women in Mathematics course to provide students with a small taste of some basic mathematics connected to the work of Lady Ada Byron Lovelace. The activity is based on her work related to the Difference Engine of Charles Babbage, and has students investigate differences of sequences. The activity could also be used in other courses, such as a general education mathematics course, a course for pre-service elementary teachers, or a history of mathematics course.

By Science & Society Picture Library, Public Domain, https://commons.wikimedia.org/w/index.php?curid=28131684

Ada Lovelace was the daughter of the famous poet Lord Byron and she lived from 1815-1852. As a mathematician she is best known for writing the first computer program which would have run on a machine designed by Charles Babbage, if it had ever been built. Babbage's Analytical Engine was to be a mechanical general purpose computer. Ada was able to visualize that the machine would be able to do more than just calculations. A mathematical physicist, Luigi Federico Menebrea, wrote an article about Babbage's Analytical Engine. Later Lady Ada Lovelace translated and greatly expanded on the article. Her translation can be found at http://www.fourmilab.ch/babbage/sketch.html . In this activity, we will investigate differences of sequences which was fundamental in Babbage's first machine, the Difference Engine, which was a step towards the Analytical Engine.

If you scroll to the fifth paragraph in Ada's translation at the webpage given above, you will see a table similar to the one below. We start with the sequence of square integers 1, 4, 9, 16, 25, … and then calculate the differences between them, called the first differences, and then calculate the differences of the differences, called the second differences.

1. Complete the table below.

Square Numbers	First Differences	Second Differences
1	*****	*****
***	3	*****
4	*****	2
*****	5	*****
9	*****	
*****		*****
16	*****	
*****		*****
25	*****	
*****		*****
	*****	
*****		*****
	*****	*****

2. What do you notice about the second differences?

3. Complete the table below.

n	n^2+n+1	First Differences	Second Differences
1	3	*****	*****
	***	4	*****
2	7	*****	2
	*****	6	*****
3	13	*****	
	*****		*****
4		*****	
	*****		*****
5		*****	
	*****		*****
6		*****	
	*****		*****
7		*****	
	*****		*****
8		*****	
	*****		*****
9		*****	*****

4. How do your results compare with the square numbers table (which could be viewed as n^2)? What do you conjecture will happen with a table of differences when computing the values of a quadratic polynomial n^2+an+b?

5. Test your conjecture above by completing the table below with integer values of your own choice for a and b in $n^2 + an + b$. Are the results as expected?

n	$n^2 + \_\_\_ n + \_\_\_$	First Differences	Second Differences
1		*****	*****
	***		*****
2		*****	
	*****		*****
3		*****	
	*****		*****
4		*****	
	*****		*****
5		*****	
	*****		*****
6		*****	
	*****		*****
7		*****	
	*****		*****
8		*****	
	*****		*****
9		*****	*****

6. Let's prove that with a polynomial of the form $n^2 + an + b$, where a and b are integers, that the second differences will be the constant 2.

a. Let k be an arbitrary integer. When the polynomial is evaluated at k, we obtain $k^2 + ak + b$. Expand the next 2 steps:

$(k+1)^2 + a(k+1) + b =$

$(k+2)^2 + a(k+2) + b =$

b. Compute the 2 first differences:

$$(k+1)^2 + a(k+1) + b - (k^2 + ak + b)$$

$$(k+2)^2 + a(k+2) + b - \left[(k+1)^2 + a(k+1) + b\right]$$

c. Using your results from b., compute the general second difference.

7. In Ada's translation and notes, as she is describing the Difference Engine, she writes, "The theorem on which is based the construction of the machine we have just been describing, is a particular case of the following more general theorem: that if in any polynomial whatever, the highest power of whose variable is *m*, this same variable be increased by equal degrees; the corresponding values of the polynomial then calculated, and the first, second, third, &c. differences of these be taken (as for the preceding series of squares); the *m*th differences will all be equal to each other." Does your work above agree with this statement? How many differences would it take for integer cubes 1, 8, 27, 64, …. ? On another sheet of paper, verify this. What was the constant and how many differences to reach it?

8. Ada describes how the dials on the Difference Engine work based on working a difference table backwards – starting with the first few values and the differences. Complete the partial table below by working backwards.

n		First Differences	Second Differences
1	5	*****	*****
	***	1	*****
2		*****	3
	*****		*****
3		*****	3
	*****		*****
4		*****	3
	*****		*****
5		*****	3
	*****		*****
6		*****	3
	*****		*****
7		*****	3
	*****		*****
8		*****	3
	*****		*****
9		*****	*****

It is interesting to note that Ada went on to explain that although a Difference Engine could only have a finite number of dials and thus only exactly compute values for polynomials of 1 less degree than the number of dials, approximations could be found for not only polynomials of higher degree but also of functions, like the logarithm, which can be represented with a convergent infinite series, e.g. a Taylor series. To read more of Lady Ada Lovelace's translation and notes on Charles Babbage's Analytical Engine, visit http://www.fourmilab.ch/babbage/sketch.html .