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Sophie Germain Activity

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Overview: This activity was originally created for a Women in Mathematics course to provide students, who may not have had an abstract algebra class, with a small taste of some basic mathematics connected to the work of Sophie Germain on Fermat's Last Theorem. The activity has the students find all primes less than 100 using the Sieve of Eratosthenes and then determine which of those are Germain primes. The activity could also be used in other courses, such as a general education mathematics course or a history of mathematics course.

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Sophie Germain was a French mathematician, born in 1776. Inspired by the story of Archimedes' death, she was determined, against her parents' wishes, to learn mathematics. She corresponded with famous mathematicians, such as Gauss and Lagrange, using the name of a male student. Her work on number theory includes an unpublished plan for tackling Fermat's Last Theorem and she won a grand prize of the French Academy for a paper on vibrating elastic surfaces in which she used a fourth order partial differential equation.

In this activity, we will investigate a small part of Sophie Germain's work in number theory connected to Fermat's Last Theorem (FLT). FLT was proposed by Pierre Fermat in 1636 and Andrew Wiles proved FLT in 1994. FLT states that there are no nontrivial integer solutions x, y, z to $x^n + y^n = z^n$ when n is a positive integer 3 or greater. Notice for n = 1 or 2 there are infinitely many nontrivial integer solutions. Solutions to the n = 2 case are called Pythagorean triples. If the exponent $n = ab$ is a composite number then if there is a solution x, y, z for n, $x^n + y^n = z^n$, there is also a solution x^a, y^a, z^a for b since $x^n + y^n = z^n$ is the same as $\left(x^a\right)^b + \left(y^a\right)^b = \left(z^a\right)^b$. This, together with the fact that Fermat proved FLT for n = 4, means that it suffices to prove FLT for all odd primes n.

1. Find all primes less than 100 using an ancient method called the Sieve of Eratosthenes. In the table below, circle 2 then cross out all multiples of 2 (except 2), then circle 3 and cross out all multiples of 3 (except 3), then 5, etc. until each number is either circled or crossed out. The circled numbers are the primes.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2. Sophie Germain was the first person to come up with a reasonable plan to prove FLT for all exponents. Before her, mathematicians tackled one exponent at a time. Part of her work on FLT involved certain prime numbers that are now named after her. A Germain prime is a prime p such that $2p + 1$ is also prime. For example, 2 is a Germain prime since both it and $2(2) + 1 = 5$ are prime. 7 is not a Germain prime since $2(7) + 1 = 15$ is not prime. (Note: It is still unknown whether or not there are infinitely many Germain primes.) Using your list of primes from #1 above, find all Germain primes less than 100.