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Ingrid Daubechies Wavelet Activity (Function Transformations)

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Ingrid Daubechies Activity

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Overview: This activity was originally created for a Women in Mathematics course to provide the students with a small taste of some basic mathematics connected to the work of Ingrid Daubechies on wavelets. The activity could also be used in a high school algebra or college algebra course to motivate transformations of functions, in particular, translation and scaling. The functions used are $f(x)=x^2$, $f(x)=e^x$, and $f(x)=\sin x$, but other functions could be used to fit the level of the students.

https://opc.mfo.de/detail?photo_id=20256 , Photo by Christoph Weber, Oberwolfach Photo Collection, Creative Commons License Attribution-Share Alike 2.0 Germany.

Ingrid Daubechies, born August 17, 1954 in Belgium, started out working in physics, but made significant contributions in mathematics. She is especially known for her work with wavelets and applications to image compression. More recently, she has been using mathematics to analyze artwork. In June of 2018, she became the first woman to win the William Bentner Prize in Applied Mathematics. (https://today.duke.edu/2018/06/duke-professor-wins-100000-prize-applied-mathematics) For this activity, you will watch a video that gives an introduction to wavelets and then investigate two concepts, scaling and shifting, mentioned in the video, when applied to familiar functions.

1. Watch the video *Understanding Wavelets, Part 1: What are Wavelets*, found at https://youtu.be/QX1-xGVFqmw . Pay special attention to what is said about scaling and shifting.

Fill in each blank with either the word "scaling" or "shifting".

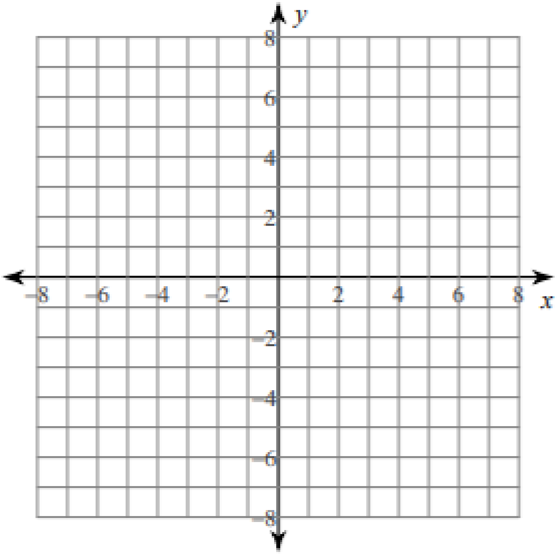
$\phi(t-k)$ ______________ Shrink ______________

$\Psi\left(\frac{t}{s}\right),\ s>0$ ______________ Translate ______________

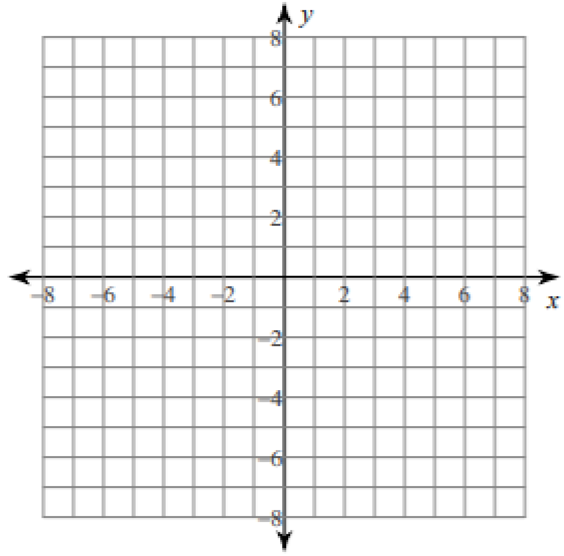
Stretch ______________ Move the center of the wavelet ______________

Graph each of the following. You may use technology, if desired.

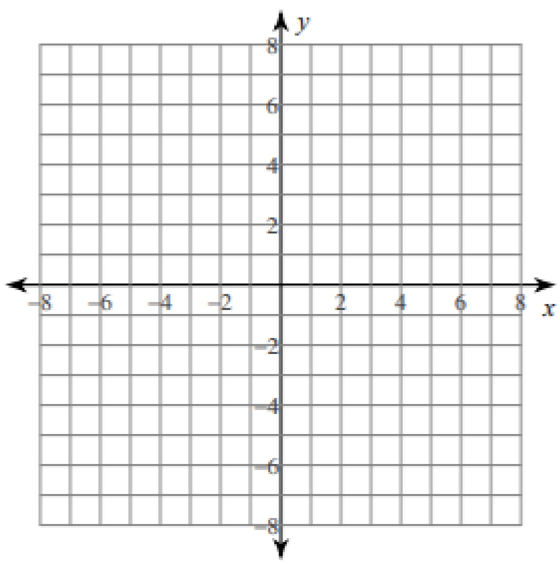
2. $f(x) = x^2$



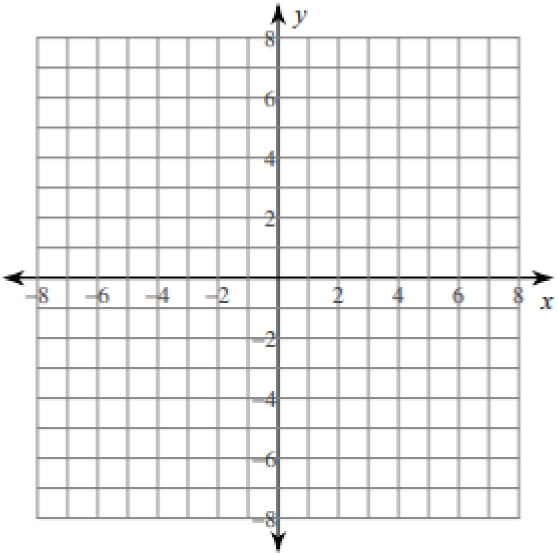
3. $f(2x) = (2x)^2 = 4x^2$



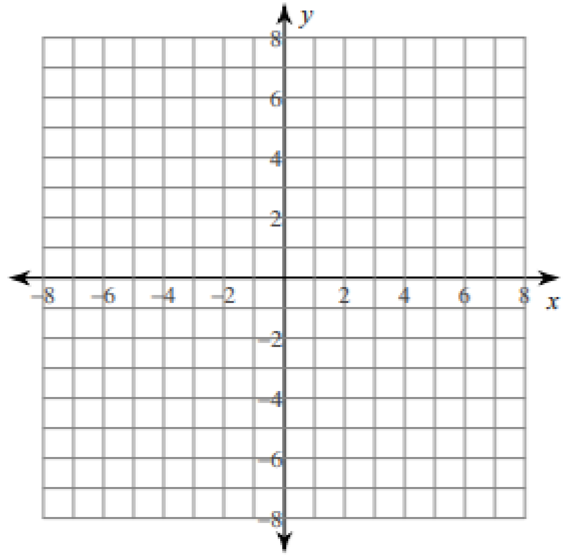
4. $f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$



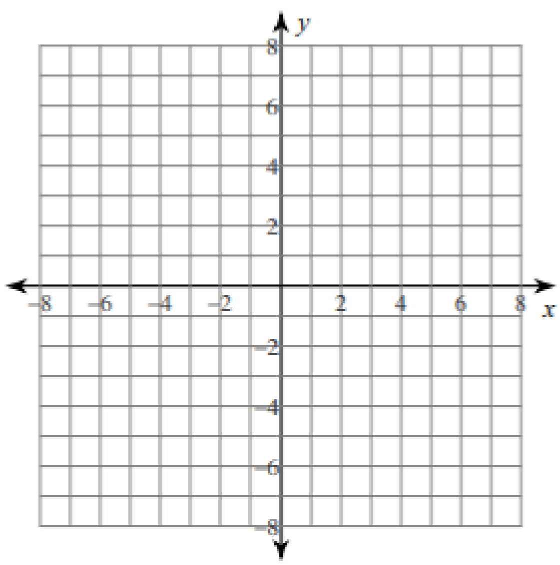
5. $f(x) = e^x$



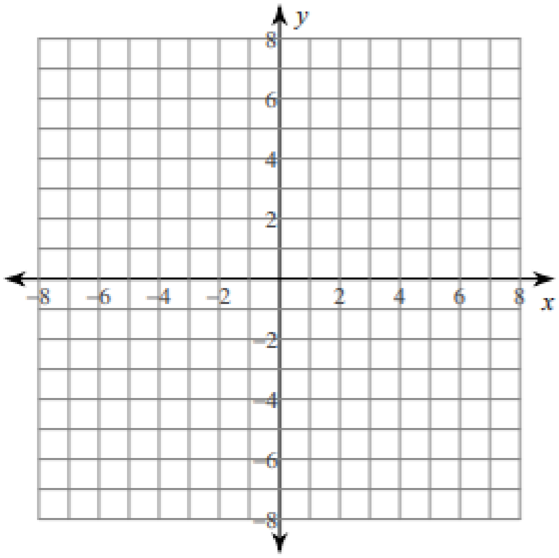
6. $f(2x) = e^{2x}$



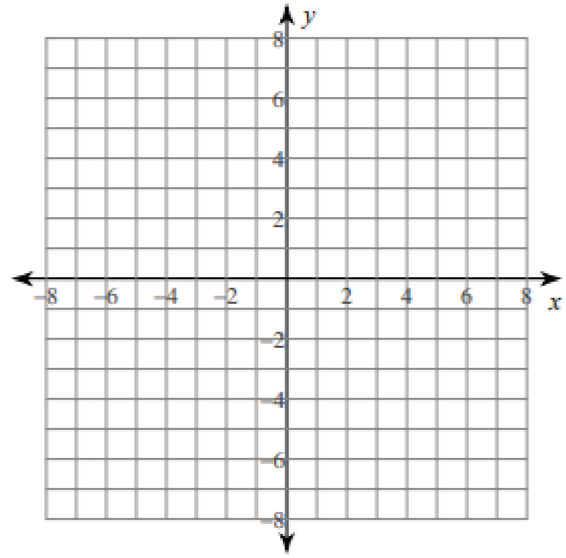
7. $f\left(\frac{1}{2}x\right) = e^{\frac{1}{2}x} = e^{\frac{x}{2}}$



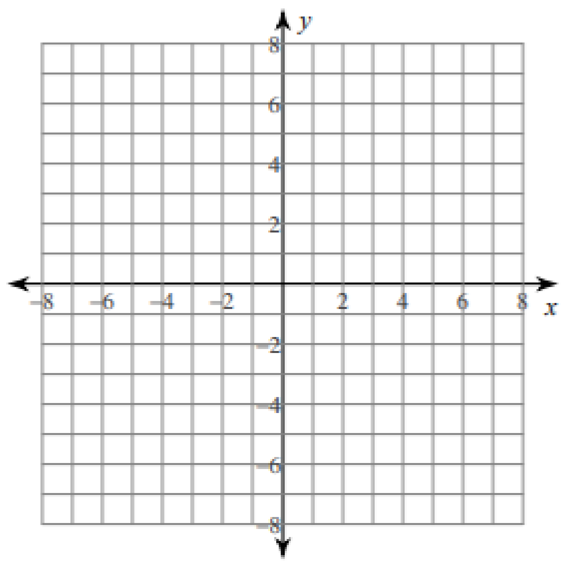
8. $f(x) = \sin x$



9. $f(2x) = \sin(2x)$

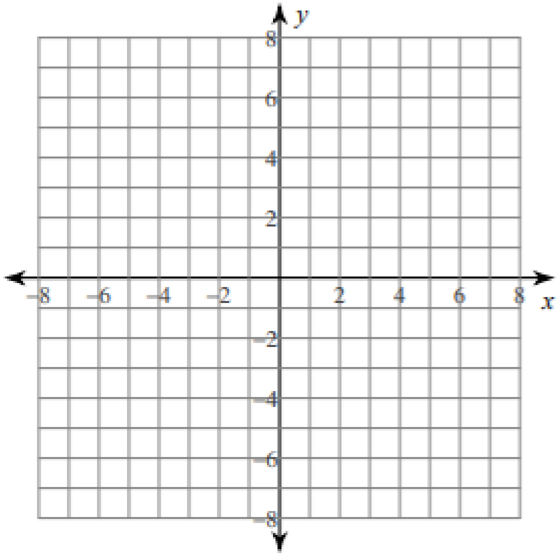


10. $f\left(\frac{1}{2}x\right) = \sin\left(\frac{1}{2}x\right) = \sin\left(\frac{x}{2}\right)$

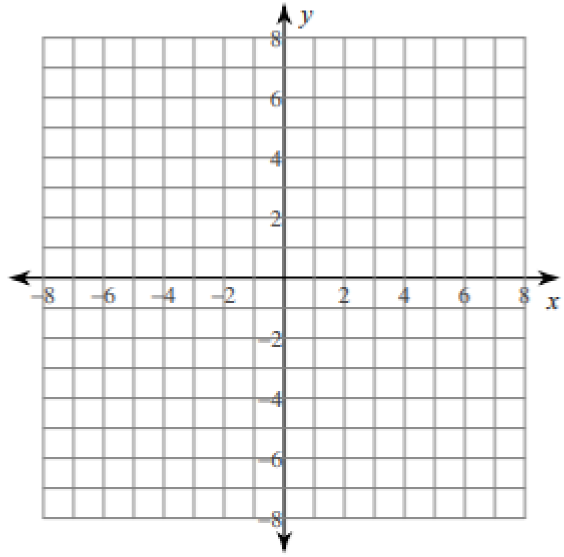


11. Based on these examples, how is the graph of $y = f(2x)$ related to $y = f(x)$? How is the graph of $y = f\left(\frac{1}{2}x\right)$ related to $y = f(x)$? What can you say in general about how the graph of $y = f(sx), s > 0$ is related to $y = f(x)$?

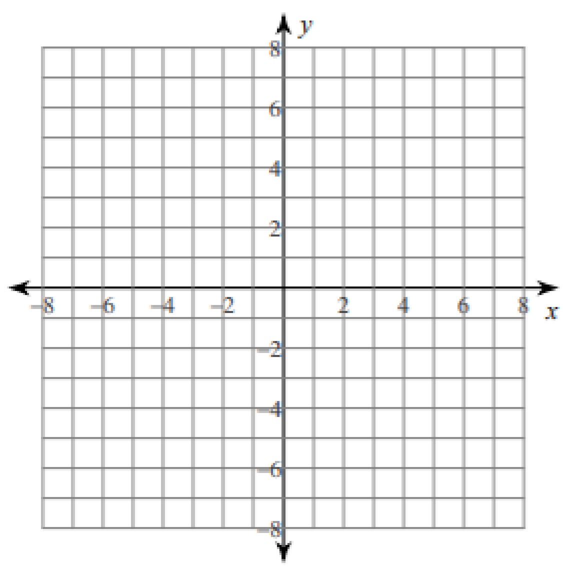
12. $f(x) = x^2$



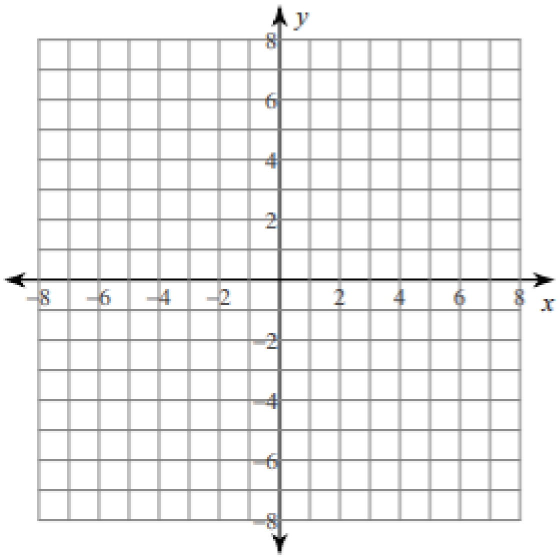
13. $f(x+2) = (x+2)^2 = x^2 + 4x + 4$



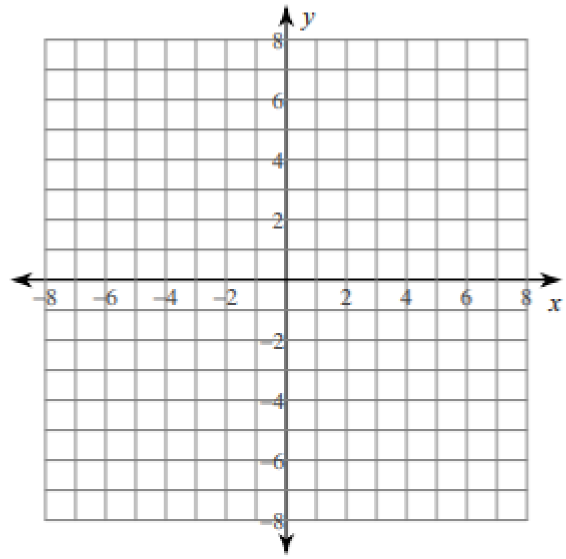
14. $f(x-2) = (x-2)^2 = x^2 - 4x + 4$



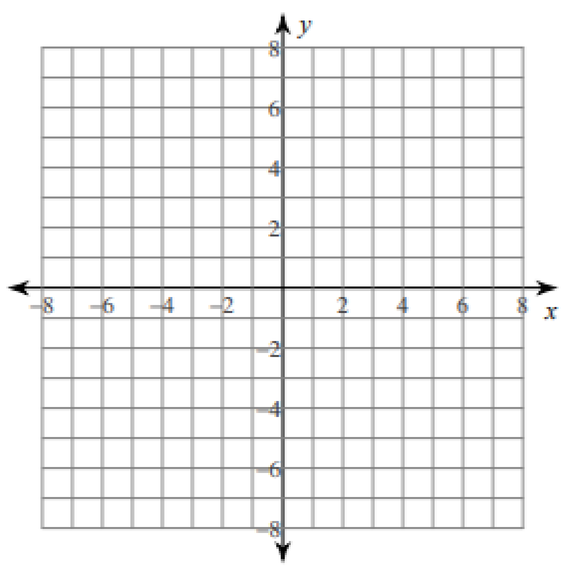
15. $f(x) = e^x$



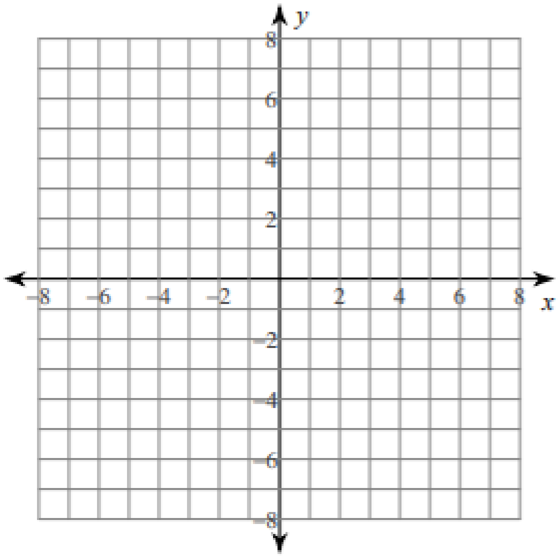
16. $f(x+2) = e^{x+2}$



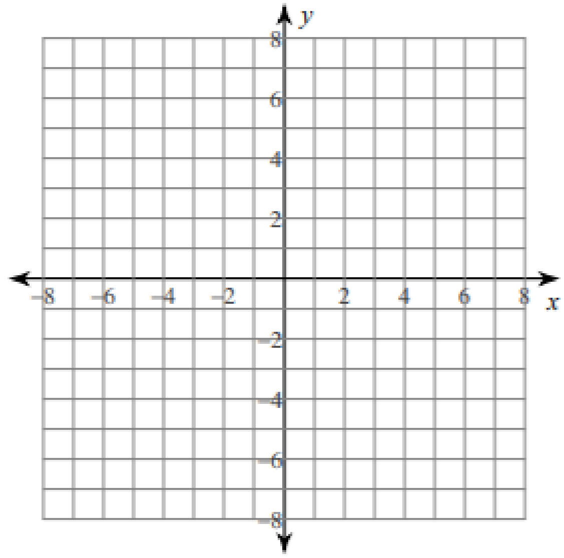
17. $f\left(x-2\right) = e^{x-2}$



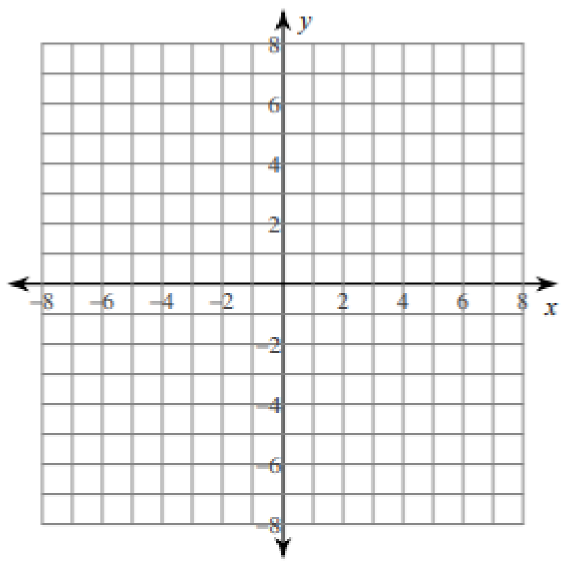
18. $f(x) = \sin x$



19. $f(x+2) = \sin(x+2)$



20. $f(x-2) = \sin(x-2)$



21. Based on these examples, how is the graph of $y = f(x+2)$ related to $y = f(x)$? How is the graph of $y = f(x-2)$ related to $y = f(x)$? What can you say in general about how the graph of $y = f(x-k)$ is related to $y = f(x)$?