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Problem Solving Practice with Problems from Fibonacci's "Liber Abbaci"

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Problem Solving with Fibonacci

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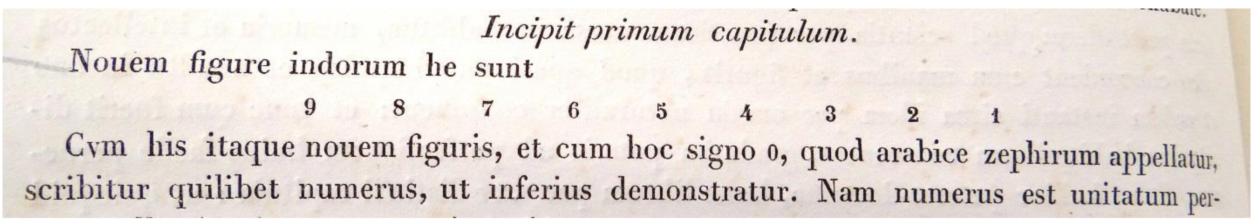
Objective: Practice problem solving skills using 2 well-known problems found in Fibonacci's *Liber Abbaci*.

Prior to the 13th century, Europeans used Roman numerals when they needed to write down numbers. Leonardo of Pisa, also known as Fibonacci, introduced the Hindu-Arabic numeral system to Europe in 1202 in his book *Liber Abacci*. They appear on page 2 at the beginning of the first chapter, where he writes



Incipit primum capitulum.

Nouem figure indorum he sunt

9 8 7 6 5 4 3 2 1

Cvm his itaque nouem figuris, et cum hoc signo 0, quod arabice zephirum appellatur, scribitur quilibet numerus, ut inferius demonstratur. Nam numerus est unitatum per-

Translated into English:

These are the nine figures of the Indians

9 8 7 6 5 4 3 2 1

And so, together with these nine figures, and with this symbol 0, which is called zephyr by the Arabs, any number can be written, which is demonstrated below.

Fibonacci then goes on to explain place value, since this is a new concept to someone only familiar with the Roman numeration system, which is mostly additive. Next he discusses how to do arithmetic with the new numeration system before giving many examples of problems with solutions. Let's try our hand at just a few. For each problem, attempt the problem yourself or with a partner or small group before looking at Fibonacci's solution. Then reflect on the similarities and/or differences in your solution as compared to Fibonacci's. Keep in mind that Fibonacci lived before the development of symbolic algebra, like we first encounter in middle school or high school. We will start with the problem for which Fibonacci is most known: The Rabbit Problem.

PROBLEM 1:

Quot paria coniculorum in uno anno ex uno pario germinentur.

Qvidam posuit unum par cuniculorum in quodam loco, qui erat undique pariete circundatus, ut sciret, quot ex eo paria germinarentur in uno anno: cum natura eorum sit per singulum mensem aliud par germinare; et in secundo mense ab eorum nativitate germinant. Quia suprascriptum par in primo mense germinat, duplicabis ipsum,

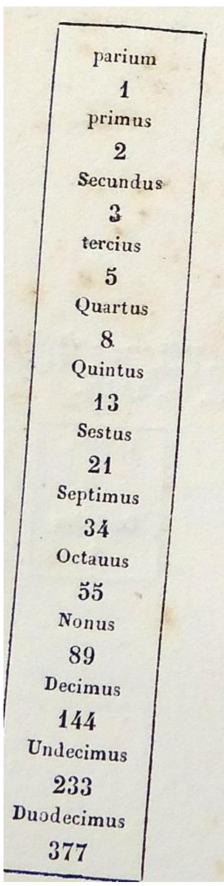
Translated into English:

How many pairs of rabbits would be born in one year from one pair.

Somebody puts one pair of rabbits in a certain place, which was surrounded on all sides by a wall, so that it may be known, how many will be born from the same pair in one year; when it is the nature of them to bear another pair during a single month; and in the second month those born of them reproduce.

Your Solution: Feel free to draw pictures.

Fibonacci's Solution: Fibonacci describes in detail for each month how many rabbits there are each month. At the start, there is one pair of rabbits. During the first month, they reproduce and there are now two pairs. In the second month, the first pair has another pair while the second pair matures, resulting in three pairs of rabbits. In the third month, two pairs reproduce and the total is now 5 pairs. Fibonacci continues to explain how many rabbits there are each month, resulting in 377 by the end of the year. He also includes a table, shown below, and mentions that adding the first and second number gives the third, adding the second and third give the fourth, and so on. He also states that this could be continued for an "unending number of months." This sequence of numbers 1, 2, 3, 5, 8, 13, 21, 35, …. (sometimes given as 1, 1, 2, 3, 5, 8, 13, 21, 25, … or 0, 1, 1, 2, 3, 5, 8, 13, 21, 25, …) is now known as the Fibonacci sequence, and the numbers in the sequence are called Fibonacci numbers. There are many sources of information about Fibonacci numbers, such as https://www.mathsisfun.com/numbers/fibonacci-sequence.html .


parium
1
primus
2
Secundus
3
tercius
5
Quartus
8
Quintus
13
Sestus
21
Septimus
34
Octauus
55
Nonus
89
Decimus
144
Undecimus
233
Duodecimus
377

Reflection: How does your solution compare with Fibonacci's?

PROBLEM 2:

De homine qui emit aues triginta trium generum pro denariis 30.

Qvidam emit aues 30 pro denariis 30. In quibus fuerunt perdices, columbe, et passeres : perdices uero emit denariis 3; columba denariis 2, et passeres 2 pro denario 1, scilicet passer 1 pro denariis $\frac{1}{2}$. Queritur quot aues emit de unoquoque genere: diuide

Translated into English:

About a man who buys 30 birds of 3 kinds for 30 denarii.

Somebody buys 30 birds for 30 denarii. Among these there are partridges, doves, and sparrows: truly he buys a partridge for 3 denarii; a dove for 2 denarii, and 2 sparrows for 1 denarius, certainly 1 sparrow for ½ denarius. Determine how many birds he buys of each kind.

Your Solution: Feel free to draw pictures.

Fibonacci's Solution: Fibonacci points out that 30 divided by 30 is 1 (he doesn't use the terminology "average" but we are averaging 1 bird for each denarius), the amounts paid per bird are ½, 2, and 3, and that the number of each type of bird will be a whole number. If we look at sparrows and partridges, we can get 5 birds (1 partridge and 4 sparrows) for 5 denarii. If we look at sparrows and doves, we can get 3 birds (1 dove and 2 sparrows) for 3 denarii. So, we need a combination of 5 and 3 that sums to 30, that is in modern terms, to find positive integers m and n with $5m + 3n = 30$. Fibonacci gives the solution of $m = 3$ and $n = 5$. So, there are 3 batches of 1 partridge and 4 sparrows and 5 batches of 1 dove and 2 sparrows, giving the final solution of 3 partridges, 5 doves, and 22 sparrows.

Reflection: How does your solution compare with Fibonacci's?