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Euler Construction Activity

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Geometry Activity based on a paper by Euler

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Original sources of mathematics provide many opportunities for students to both do mathematics and to improve their problem solving skills. It is also interesting to explore original sources in new ways with the use of technology. In this activity, students can gain experience with dynamic geometry software and enhance their geometric intuition by working through a construction given by Euler in 1783.

Euler, one of the most, if not the most, prolific mathematicians of all time, wrote on a variety of subjects including mathematics. He lived from 1707 to 1783. The image of Euler on the left is from a 1753 pastel painting by Jakob Emanuel Handmann. This activity is based on a paper that was written in Latin and published in the journal *Acta Academiae Scientarum Imperialis Petropolitinae*, which was the journal of the St. Petersburg Academy. The title is *Problematis cuiusdam Pappi Alexandrini constructio*, which can be translated as *The Construction of a Certain Problem of Pappus of Alexandria.* The activity uses the English translation by Cameron Friend and Dr. Cynthia Huffman which can be found on the Euler Archive under E543. (http://eulerarchive.maa.org/)

Task 1: Read through the statement of the construction problem from Euler's article and answer the following two questions.

a. What is the given information?

b. What is the goal of the construction?

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Problem.

In a given circle, with drawn center O, to inscribe triangle *abc*, whose three sides *ab, ac, bc* having been extended, cross through the given three points *C*, *B*, *A*.

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Task 2: Read through the construction process given by Euler. Using a dynamic geometry software, such as GeoGebra, work through the steps.

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Construction.

Let A, B, C be three given points, whose distances from the center O of the circle are $AO = a, BO = b, CO = c$; let the radius of the circle = 1. Now from point B may the interval $BF = \frac{bb-1}{AB}$ be taken and it will be $FO^2 - 1 = \frac{BF(aa-1)}{AB}$. Then, with line FC having been joined, on it may be captured the interval

$$FK = \frac{FO^2 - 1}{FC} = \frac{BF(aa-1)}{AB \cdot FC}, \text{ and it will be}$$

$$KO^2 - 1 = \frac{FK(cc-1)}{FC}.$$

Now the radius Om is drawn from center O, so that *the cosine of angle* $KOm = \frac{\cos BFC}{KO}$. Then let angle BFC be bisected by line segment FS, parallel to which line segment mb is drawn from point m, and b will be one of the angles of the desired triangle, to which if from point A line segment Ab[1] is drawn, which having been extended will cut the circle at c[2]. From this point c towards B the line segment cB is drawn cutting the circle at a; then certainly the side ba having been extended will cross through the third given point C, and it will result in abc the desired triangle.

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Task 3: Reflect on your construction. Were you able to construct the desired triangle? Were the steps easy to follow? Were any assumptions made? Euler included a figure in his article which is on the next page. Did your finished product look similar to the figure from Euler's article?

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[1] This is uppercase in the original document, but clearly referring to the lowercase b it references.
[2] In the original text, this is an uppercase C, but we've changed it here and in the instances which follow.

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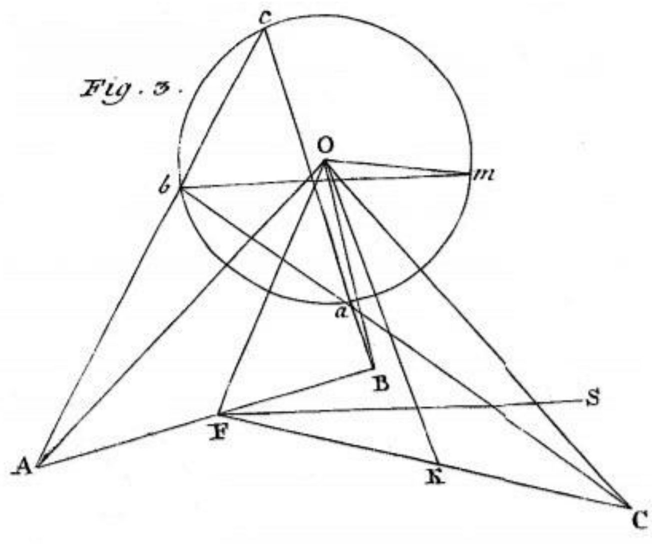

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Task 5: Below is Corollary 1 from Euler's article, which is another construction. It involves drawing a line parallel to a given line through a point. How can this be accomplished using your dynamic geometry software?

Task 6: Work through the construction given in Corollary 1 using dynamic geometry program. Were you able to construct the desired triangle? Were the steps easy to follow? Were any assumptions made? Did your finished product look similar to the figure from Euler's article? What do you think Euler meant when he mentioned points being infinitely separated? How does the construction compare to your first construction.

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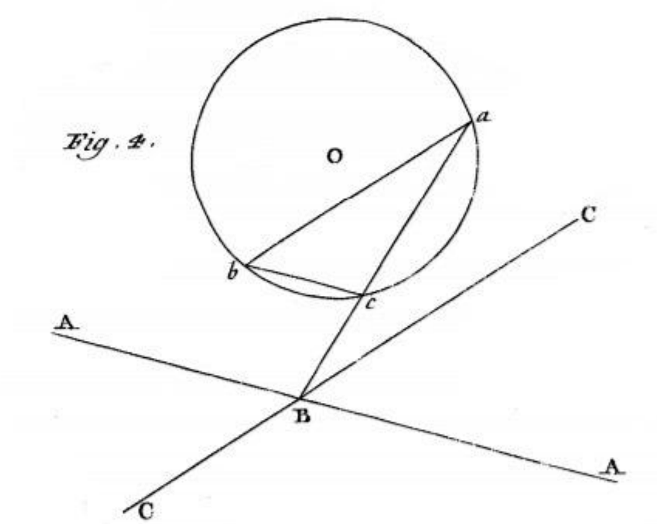
Corollary 1.

Let two of the given points *A* and *C* be infinitely separated on the lines *ABA*, *CBC* crossing one another at *B*. From *B* the line *Bca* is drawn, cutting off an arc *ca* from the circle, on which angles may be set on the circumference, equals to angle *CBA*, then drawing from *a* the line *ab* parallel to *CBC* itself, and *bc* parallel to *ABA* itself, *abc* will be the desired triangle.



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Task 7: Work through the construction given in Corollary 2 using dynamic geometry program. Were you able to construct the desired triangle? Were the steps easy to follow? Were any assumptions made? Did your finished product look similar to the figure from Euler's article? What do you think Euler meant when he mentioned points being infinitely separated? How does the construction compare to the previous two constructions.

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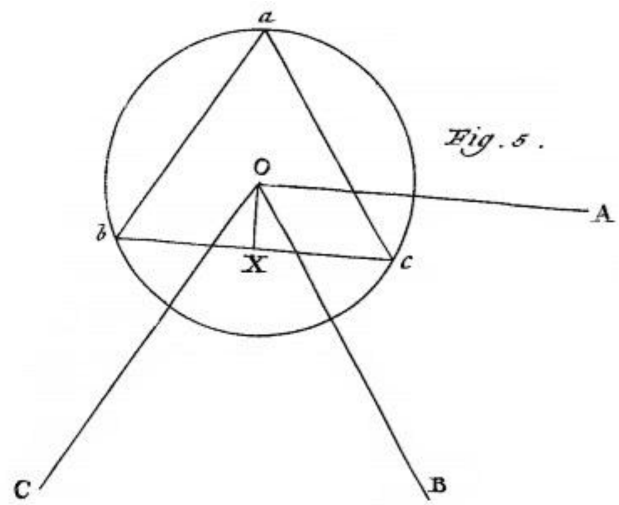
Corollary 2.

Let all three given points fall at infinite distances on the lines *OA*, *OB*, *OC*; next the line *bc* may be drawn parallel to line *OA* at a distance from the center *OX*= cos *BOC*; then the desired triangle will have line *ba* having been drawn parallel to *OC* itself and *ca* parallel to *OB* itself.



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Task 8: Can you think of any applications of any of the three constructions?

Task 9: Pick one of the constructions and write out instructions in your words for completing the construction.

Task 10: Conclusions? Questions?