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# A Historical Activity Adding Irrational Numbers

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# A Historical Activity on Adding Certain Irrational Numbers

by

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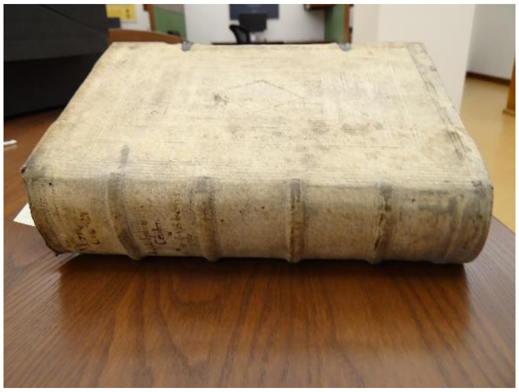
## Description

In this activity, students will follow and then analyze work by mathematician and fencing master Ludolph Van Ceulen (1540-1610) on adding similar radical numbers, like $2\sqrt{3}+5\sqrt{3}$. His method is quite different from the method currently in use in the secondary school curriculum and College Algebra, in that he does not first simplify the square root expressions before adding them. This activity could be used with high school or College Algebra students after they have learned the current method, in order to reinforce their understanding of radicals. The activity would also be appropriate to use with pre-service mathematics teachers or in a history of math class or in a math circle. It demonstrates that there may be more than one algorithm or technique for performing arithmetic operations, and that mathematics, including arithmetic, develops and evolves over time.

## Historical Background

We will be analyzing a primary source that was published in 1619 in Latin. *Surdorum quadraticorum arithmetica* (Arithmetic of Square Roots) is a work by the mathematician and fencing master Ludolph Van Ceulen (1540-1610), which was translated into Latin by his student Willebrord Snell (1580-1626). It was published after the death of Van Ceulen, along with the Snell's modification and Latin translation *De Circulo* of Van Ceulen's *Vanden Circkel*, which was published in 1596. Below are two pictures of the book at the Linda Hall Library in Kansas City, MO.

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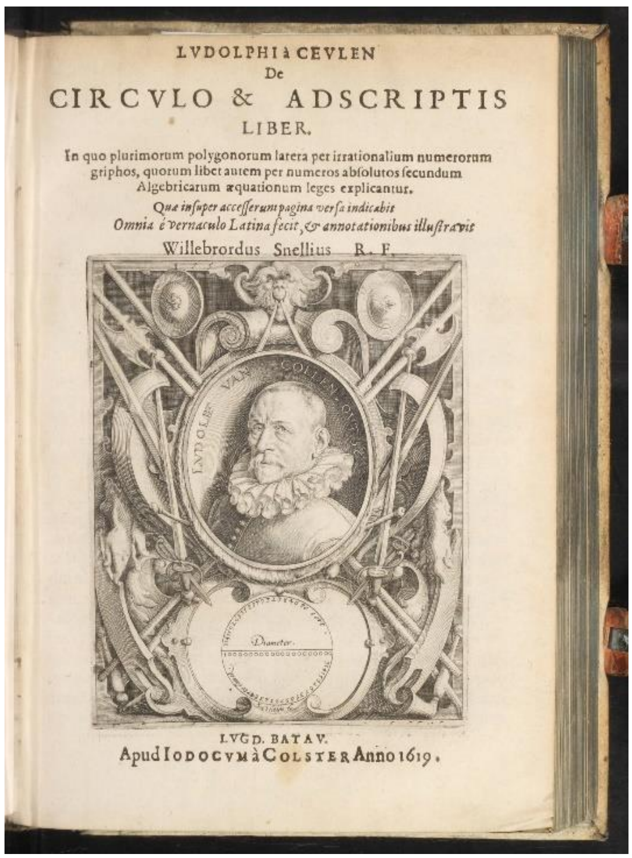
Van Ceulen's parents were not wealthy and had a large family to support, so Ludolph did not receive a university education. Although born in Germany, as an adult, Van Ceulen lived in the Netherlands. Beginning about the time he was 26, he made his living teaching mathematics. Van Ceulen had 5 children with his first wife before she died and then he married a woman with 8 children. When he was 40, he opened a fencing school in Delft. So, in addition to being a mathematician, he was also a fencing master. A few years later Archimedes' method of approximating $\pi$ was translated from the Greek for him, and Van Ceulen proceeded to use the technique to improve on approximations of $\pi$, publishing *Vanden Circkel* in 1596 in which he approximated $\pi$ to 20 decimal places. The title page of the 1619 Latin edition of Van Ceulen's *De Circulo* is below.


LVDOLPHI à CEVLEN
De
CIRCVLO & ADSCRIPTIS
LIBER.
In quo plurimorum polygonorum latera per irrationalium numerorum griphos, quorum libet autem per numeros absolutos secundum Algebricarum æquationum leges explicantur.
Quæ insuper accesserunt pagina versa indicabit
Omnia è vernaculo Latina fecit, & annotationibus illustravit
Willebrordus Snellius R. F.

LVGD. BATAV.
Apud IODOCVM à COLSTER Anno 1619.

The engraving on the title page is the same one used for the 1596 Dutch original. Notice that below the portrait of Van Ceulen is a circle with diameter of $10^{20}$. Along the top and bottom semicircles is printed 314159265358979323846 te cort (too short) and 314159265358979323847 te lanck (too long). These bounds for $\pi$ were the best ones available when *Vanden Circkel* was published in 1596.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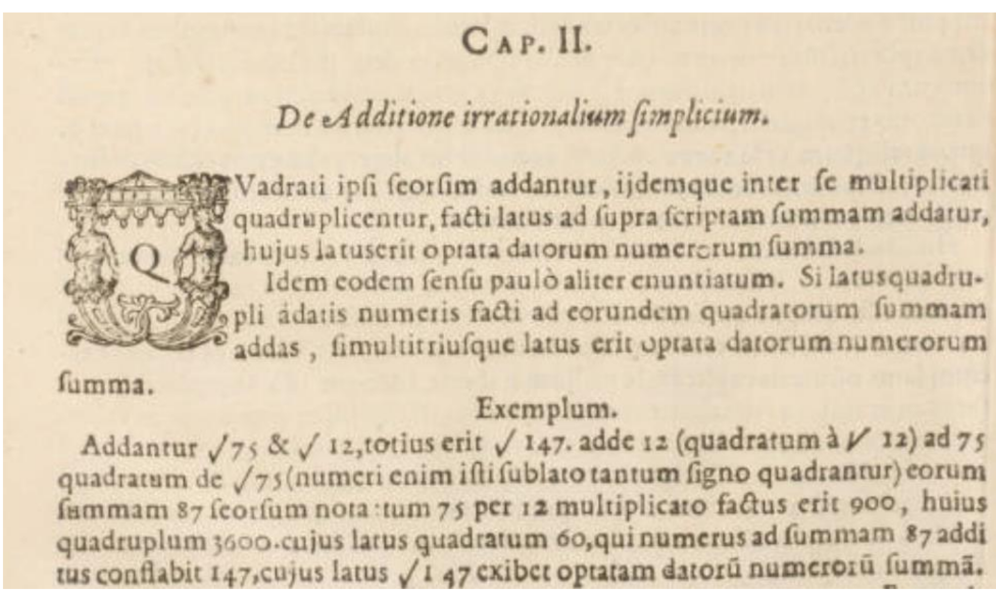
Later Van Ceulen would determine $\pi$ to 35 decimal places. As evidence of the importance of Van Ceulen's work, $\pi$ has been known as "Ludolph's number" or the "Ludolphine number" in Germany and the Netherlands.

Although Van Ceulen is known for his part in the story of $\pi$, he also wrote other works. We will now take a look at the Latin translation of one of these works, called *Surdorum quadraticorum arithmetica* (Arithmetic of Square Roots).

## Activity

Chapter 2 is about the Addition of Simple Irrational Numbers.



CAP. II.

*De Additione irrationalium ſimplicium.*

Quadrati ipſi ſeorſim addantur, ijdemque inter ſe multiplicati quadruplicentur, facti latus ad ſupra ſcriptam ſummam addatur, hujus latus erit optata datorum numerorum ſumma.

Idem eodem ſenſu paulò aliter enuntiatum. Si latus quadrupli à datis numeris facti ad eorundem quadratorum ſummam addas, ſimul utriuſque latus erit optata datorum numerorum ſumma.

Exemplum.

Addantur √75 & √12, totius erit √147. adde 12 (quadratum à √12) ad 75 quadratum de √75 (numeri enim iſti ſublato tantum ſigno quadrantur) eorum ſummam 87 ſeorſum nota: tum 75 per 12 multiplicato factus erit 900, huius quadruplum 3600. cujus latus quadratum 60, qui numerus ad ſummam 87 additus conflabit 147, cujus latus √147 exibet optatam datorũ numerorũ ſummã.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For his first example, Van Ceulen states that $\sqrt{75}+\sqrt{12}=\sqrt{147}$ .

**TASK 1:** a) Does this solution seem obvious to you? YES or NO

b) Does this solution seem reasonable to you at first glance? YES or NO

c) Check if it is reasonable that $\sqrt{75}+\sqrt{12}=\sqrt{147}$ by using a calculator.

$\sqrt{75} \approx$ ___________ $\sqrt{12} \approx$ ___________

$\sqrt{75}+\sqrt{12} \approx$ ___________ $\sqrt{147} \approx$ ___________

d) Verify $\sqrt{75}+\sqrt{12}=\sqrt{147}$ by simplifying the radicals and then adding.

**TASK 2:** Van Ceulen goes on to explain his method for adding $\sqrt{75}+\sqrt{12}$. Follow along by filling in the blanks.

a) First, add $75+12=$ ___________ .

b) Next, compute the product $75\times 12=$ ___________

c) Multiply the product from part b) by 4. ______________

d) Find the square root of your solution to part c). __________

e) Add your answer to part a) with your answer to part d) . __________

f) Finally, write your answer to part e) under a radical, $\sqrt{75}+\sqrt{12}=$ ___________

g) Did Van Ceulen's method give the correct answer? YES or NO

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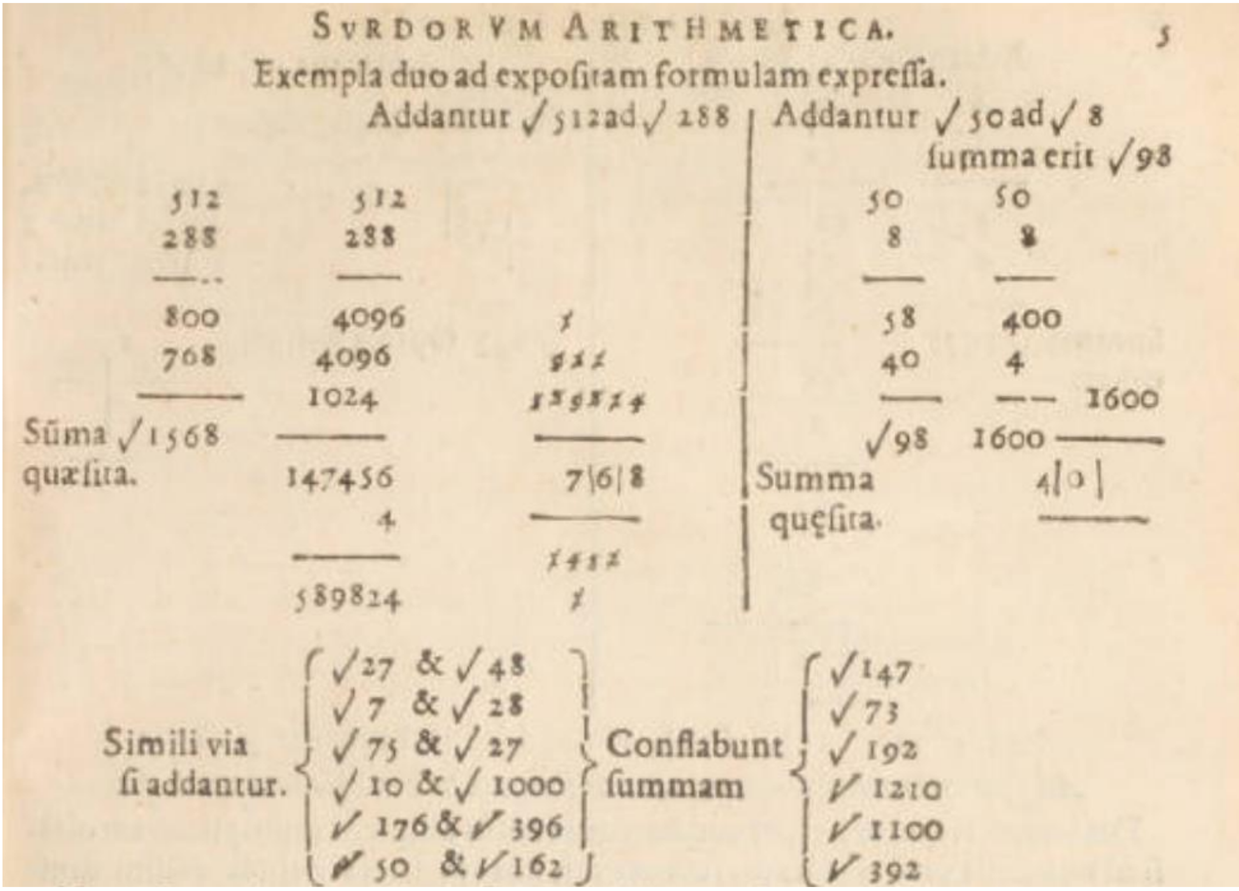

SVRDORVM ARITHMETICA. 5

Exempla duo ad expositam formulam expressa.

Addantur √512 ad √288 | Addantur √50 ad √8 summa erit √98

| | | |
|---|---|---|
| 512 | 512 | |
| 288 | 288 | |
| 800 | 4096 | |
| 768 | 4096 | |
| | 1024 | |
| Sũma √1568 quæsita. | 147456 | 7\|6\|8 |
| | 4 | |
| | 589824 | |

| | | |
|---|---|---|
| 50 | 50 | |
| 8 | 8 | |
| 58 | 400 | |
| 40 | 4 | |
| | | 1600 |
| √98 | 1600 | |
| Summa quęsita. | | 4\|0\| |

Simili via si addantur: √27 & √48, √7 & √28, √75 & √27, √10 & √1000, √176 & √396, √50 & √162 — Conflabunt summam: √147, √73, √192, √1210, √1100, √392

**TASK 3:** Above is an image from page 5 of Van Ceulen's *Surdorum Arithmetica* with several examples (2 worked out and 6 with just the solution) of adding similar radicals. Pick one of these and carry out the addition using Van Ceulen's method by filling in the blanks below.

a) First, add the radicands _____ + _____ = ________.

b) Next, compute the product ______× ______ = ___________

c) Multiply the product from part b) by 4. ______________

d) Find the square root of your solution to part c). __________

e) Add your answer to part a) with your answer to part d) . __________

f) Finally, write your answer to part e) under a radical,
$\sqrt{\_\_\_\_\_\_} + \sqrt{\_\_\_\_\_\_} = \sqrt{\_\_\_\_\_\_\_\_\_\_\_\_}$

g) Did Van Ceulen's method give the correct answer? YES or NO

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TASK 4: Make up your own example (be sure that the radicals you pick are similar when simplified) and carry out the addition using Van Ceulen's method by filling in the blanks below.

a) The example is $\sqrt{\_\_\_\_\_} + \sqrt{\_\_\_\_\_}$

b) First, add the radicands _____ + _____ = ________.

c) Next, compute the product of the radicands ______ $\times$ ______ = ___________

d) Multiply the product from part c) by 4. ______________

e) Find the square root of your solution to part d). __________

f) Add your answer to part b) with your answer to part e) . __________

g) Finally, write your answer to part f) under a radical,
$\sqrt{\_\_\_\_\_} + \sqrt{\_\_\_\_\_} = \sqrt{\_\_\_\_\_\_\_\_\_\_}$

h) Did Van Ceulen's method give the correct answer? YES or NO

TASK 5: Now that we have seen that Van Ceulen's method works for several examples, let us prove that it will always work by applying it to the general case $a\sqrt{c} + b\sqrt{c}$, where $a, b, c$ are positive integers. Before we get started, we need each term in the sum written as Van Ceulen would have them. So, we need each term written as the square root of a single number, not as an integer multiple of a square root.

a) The general case, in the form Van Ceulen would have used, is $\sqrt{\_\_\_\_\_} + \sqrt{\_\_\_\_\_}$, where $a, b, c$ are positive integers.

b) First, add the radicands _____ + _____

c) Next, compute the product of the radicands $a^2c \times$ ______ = ___________

d) Multiply the product from part c) by 4. ______________

e) Find the square root of your solution to part d). __________

f) Add your answer to part b) with your answer to part e) . __________

g) Finally, write your answer to part f) under a radical,
$\sqrt{\_\_\_\_\_} + \sqrt{\_\_\_\_\_} = \sqrt{\_\_\_\_\_\_\_\_\_\_} = (\_\_\_\_ + \_\_\_\_)\sqrt{c} = a\sqrt{c} + b\sqrt{c}$

h) Did Van Ceulen's method give the correct answer? YES or NO


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REFERENCES:

- Ludolph Van Ceulen, MacTutor History of Mathematics Archive, http://mathshistory.st-andrews.ac.uk/Biographies/Van_Ceulen.html

- *Surdorum quadraticorum arithmetica* (Arithmetic of Square Roots), digital scan, Linda Hall Library, http://lhldigital.lindahall.org/cdm/ref/collection/math/id/14975

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